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Optimal Planning of Accelerated Reliability Demonstration Tests for the Weibull Model

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Demonstrating product reliability is challenging, which is why accelerated tests are commonly used to reduce time and cost. The Probability of Test Success (P_{ts}) helps assess a test's ability to demonstrate the reliability requirement under given conditions, representing the statistical power of the accelerated reliability demonstration test. Key challenges remain, particularly selecting the number of specimens, load intensities, and number of load levels to maximize P_{ts}while minimizing cost. Although previous work has supported risk-based planning, recent analytical advances in P_{ts} formulation offer more refined optimization methods.
This paper introduces a new approach that provides a simple, fast algorithm for determining optimal accelerated test settings based on product performance and reliability requirements. A notable novelty is the integration of the actual load-profile shape, which influences reliability demonstration due to Type I statistical error. The method incorporates the widely used lifetime models of a Wöhler-Weibull model and includes a brief study illustrating its practical advantages.

Keywords: Reliability Testing, Accelerated Life Testing, Life Data Analysis, Reliability Modeling.

1. Introduction

Demonstrating the reliability of modern engineering systems remains a fundamental requirement in product development [1], [2], yet achieving statistically sound reliability demonstration within acceptable time and cost constraints continues to be a major challenge [3], [4], [5]. Accelerated testing therefore plays a critical role, as it enables engineers to evaluate long-term performance under elevated loads and thereby infer field reliability more efficiently [6], [7]. However, despite its widespread use, the planning of accelerated reliability demonstration tests is far from trivial.

Several methodological advances—most notably the formalization of the Probability of Test Success (P_{ts}) [3], [4], [8], [9], [10], [11], [12], [13], [14], [15]—have significantly strengthened the analytical foundation of test planning. P_{ts} provides a quantitative measure of a test's capability to demonstrate a specified reliability requirement and thus supports more holistic and risk-aware decision making. Yet current applications typically rely on forward-evaluation given a predefined set of test parameters, one computes the resulting P_{ts}. While valuable, this approach does not directly answer the central planning question faced by practitioners: Which combination of load levels, specimen allocation, and test conditions yields the highest possible P_{ts} within given boundary conditions e.g. on time and budget. For example, the recent work by Mell and Dazer [16] make use of some very valuable analytic equations in this context and allow for a thorough analysis of test settings, although using a different approach for the P_{ts} calculation. Also Herzig et al. [14], [17] conducted some insightful research on the influences of the boundary conditions and test settings for a successful reliability demonstration. Benz et al. additionally investigated the influence of different load profile shapes [18].

The objective of this work is to address the still remaining gap of forward-evaluation versus to be know test settings. Building upon established formulations of P_{ts} in the hypothesis testing framework [10], [11], [12], [13], [18], [19], [20], [21], [22], we propose an optimization-based framework that leverages analytically derived, novel expressions to determine the optimal accelerated test configuration. By integrating these analytical expressions into a suitable optimization algorithm, the method identifies test settings that maximize P_{ts} considering the underlying lifetime model, product characteristics, and reliability requirements as well as boundary conditions of expenditure (e.g. maximum sample size). This marks a conceptual shift from evaluation to optimization and enables a more systematic, efficient, and statistically justified approach to accelerated test planning for reliability demonstration. The models used herein are the Weibull distribution [23] and the Wöhler model [24] as the underlying lifetime model.

2. Probability of Test Success for Accelerated Reliability Demonstration Tests

Reliability demonstration tests are commonly designed as success-run tests because the underlying binomial formulation enables straightforward sample size determination. However, this classical approach accounts only for Type I statistical error and therefore does not quantify a test's capability to successfully demonstrate the reliability requirement. To address this limitation, the Probability of Test Success (P_{ts}) was introduced in prior work [3], [4], [11], [13], [18]. P_{ts} interprets the reliability demonstration test as a hypothesis test on the lifetime quantile t_R for required reliability R_{req}, comparing the required service life t_{req} with its estimated value. The resulting hypotheses are

$$H_0\colon\ t_R < t_{\mathrm{req}} \tag{1}$$

$$H_1\colon\ t_R \geq t_{\mathrm{req}} \tag{2}$$

The method evaluates two distributions: the null distribution f_{H_0} under the assumption that the required service life is not met (H_0), and the alternative distribution f_{H_1} under the assumption that it is satisfied (H_1). By integrating these distributions at a critical threshold chosen to meet the confidence requirement, the probability of achieving a successful demonstration can be computed.

$$P_{\mathrm{ts}} = \int_{t_{\mathrm{crit}}}^{\infty} f_{H_1}(t_R)\,\mathrm{d}t_R \tag{3}$$

$$C = \int_0^{t_{\mathrm{crit}}} f_{H_0}(t_R)\,\mathrm{d}t_R \tag{4}$$

Various approaches—ranging from Monte-Carlo-based derivations to asymptotic likelihood-based approximations—have been developed to calculate these distributions for success-run, failure-based, censored, accelerated, and system-level tests [10], [11], [12], [13], [19], [20], [21], [22]. These methodological foundations form the basis for applying P_{ts} in the context of optimal accelerated test planning for reliability demonstration.

2.1. *Hypothesis Testing for Reliability using Lifetime Model Estimation in Accelerated Tests*

As with all tests for reliability demonstration, the lifetime quantile needs to be estimated according to required reliability and confidence. In the context of accelerated testing, the failure times at the respective load levels are used to estimate the lifetime model in combination with the failure distribution parameters [1], [7]. These are then used to estimate the lifetime based on a damage accumulation model [24].

In this paper the widely used Wöhler-Weibull model is used [22]. The failure distribution is

$$f_{WW}(t) = \frac{b}{T}\left(\frac{t}{T}\right)^{b-1} \cdot e^{-\left(\frac{t}{T}\right)^{b}} \tag{5}$$

with

$$T = N_A \cdot \left(\frac{S}{S_A}\right)^{-k} \cdot (-\ln(0.5))^{-\frac{1}{b}}. \tag{6}$$

The parameters of the model are N_A, the endurable cycles of the lifetime at fixed load height S_A, k the slope of the model and b the Weibull shape parameter. The scale parameter T depends on the load level height S. The factor $(-\ln(0.5))^{-\frac{1}{b}}$ is used so that the parameters correspond to a 50 % Wöhler-Curve instead of a 62.3 % as it would be dedicated by the structure of the Weibull model.

A lifetime quantile t_R for reliability R is calculated by miners rule [24], [25] and the load profile of interest with M load levels S_j and corresponding cycle counts N_j

$$t_R = \frac{D_{\mathrm{crit}}}{D_{\mathrm{profile}}} = \frac{N_A\left(-\frac{\ln(R)}{\ln(0.5)}\right)^{\frac{1}{b}}}{\sum_{j=1}^{M} N_j\left(\frac{S_j}{S_A}\right)^{k}}. \tag{7}$$

The accelerated test uses two or three load levels S_I, S_{II} and S_{III} to gather failure times t_i on those load levels with respective sample size n_I, n_{II} and n_{III}. The parameters of the model $\Theta = [N_A, k, b]$ are estimated by Maximum-Likelihood estimation (MLE) [2], [7] and are used with eq. (7) to calculate the estimated lifetime quantile.

For calculating the P_{ts}, the null and alternative distributions of t_R need to be estimated. This can be done be employing the bootstrap approach introduced in [22], [26]. It samples failure times according to test settings (S, n) repeatedly to gather several estimates of Θ which propagate using eq. (7) to obtain several values of t_R. This is done for validity of H_1 to get the alternative distribution and under validity of H_0 to get an empiric estimate for the null distribution. For this, prior knowledge about the parameters of the model Θ need to be known or at least be decided upon to be relevant of interest for the analysis.

2.2. *Calculating the Probability of Test Success using Expected Variance and Hypothesis Testing Approach*

Already Grundler et al. [11], [13], [19], [21], [26] introduced analytic equations to enable a quick and less computational way of calculating the P_{ts} of non-accelerated tests by making use of asymptotic properties. The established hypothesis testing framework helped in this regard, since effectively only one distribution must be estimated in contrast to the initial Monte-Carlo simulation based approach by Dazer et al. [3], [4], [11], [13], [19], [21].

Although using the initial approach of simulating the estimation process in contrast to following a hypothesis testing framework, Mell and Dazer introduced the expected fisher information matrix for calculating successful reliability demonstration probabilities of accelerated tests [16]. This will be used in the following to derive analytical equations for accelerated reliability demonstration tests in the context of a hypothesis testing framework, so that the P_{ts} correctly represents the type II statistical error or power of the reliability test.

The Log-Likelihood of the model is

$$\Lambda(N_A, k, b) = n \cdot \ln(b) - b\sum_{i=1}^{n} \ln\left(N_A\left(\frac{S_i}{S_A}\right)^{-k}\right) + (b-1)\sum_{i=1}^{n}\ln(t_i) - \sum_{i=1}^{n}\left(\frac{t_i}{N_A\left(\frac{S_i}{S_A}\right)^{-k}}\right)^{b}. \tag{8}$$

Here, the summation index i is over all $n = n_I + n_{II}(+n_{III})$ samples and their corresponding load levels S_I. By setting $Z_i = \frac{t_i}{N_A\left(\frac{S_i}{S_A}\right)^{-k}}$, and $\frac{\partial \Lambda}{\partial N_A} = 0$ which results in $\sum_{i=1}^{n} Z_i^b = n$, the variable Z_i is following a Standard-Weibull distribution with $T = 1$. The second partial derivatives of Λ with respect to Θ results in terms dependent on Z^b, $Z^b\ln(Z)$ and $Z^b(\ln(Z))^2$ whose expectations are

$$\mathrm{E}[Z^b] = 1 \tag{9}$$

$$\mathrm{E}[Z^b \ln(Z)] = \frac{1-\gamma}{b} \tag{10}$$

$$\mathrm{E}[Z^b \ln(Z)^2] = \frac{(1-\gamma)^2 + \frac{\pi^2}{6}}{b^2}. \tag{11}$$

See also [16] for a more detailed derivation of the equations. The expected Fisher-Information matrix I finally results in

$$\mathrm{I} = \begin{bmatrix} n\left(\frac{b}{N_A}\right)^2 & -\frac{b^2}{N_A}\sum_{i=1}^{n}\ln\left(\frac{S_i}{S_A}\right) & -\frac{n}{N_A}(1-\gamma) \\ \vdots & b^2\sum_{i=1}^{n}\left(\ln\left(\frac{S_i}{S_A}\right)\right)^2 & (1-\gamma)\sum_{i=1}^{n}\ln\left(\frac{S_i}{S_A}\right) \\ \text{sym.} & \cdots & \frac{n}{b^2}\left((1-\gamma)^2 + \frac{\pi^2}{6}\right) \end{bmatrix}. \tag{12}$$

Since the expected matrix is used here, no synthetic failure times need to be estimated as in the initial approach presented in [22]. However, the core concept is the same, with the exception of using the expected matrix presented in [16]. The Variance-Covariance matrix is the inverse of the Fisher-Information matrix: $V = I^{-1}$.

The derivatives of the quantile function of eq. (7) are

$$\frac{\partial \mathrm{t_R}}{\partial N_A} = \frac{\left(-\frac{\ln(R)}{\ln(0.5)}\right)^{1/b}}{\sum_{j=1}^{M} N_j\left(\frac{S_j}{S_A}\right)^{k}} \tag{13}$$

$$\frac{\partial \mathrm{t_R}}{\partial k} = \frac{N_A\left(-\frac{\ln(R)}{\ln(0.5)}\right)^{1/b}}{\sum_{j=1}^{M} N_j\left(\frac{S_j}{S_A}\right)^{k}}\ln\left(\frac{S_j}{S_A}\right) \tag{14}$$

$$\frac{\partial \mathrm{t_R}}{\partial b} = \frac{N_A\left(-\frac{\ln(R)}{\ln(0.5)}\right)^{1/b}}{b^2\sum_{j=1}^{M} N_j\left(\frac{S_j}{S_A}\right)^{k}}\ln\left(-\frac{\ln(R)}{\ln(0.5)}\right). \tag{15}$$

With these, the variance of the lifetime quantile of interest $\mathrm{Var}(t_R)$ is calculated as

$$\mathrm{Var}(t_R) = \nabla\Theta^T V \nabla\Theta = \begin{bmatrix} \frac{\partial \mathrm{t_R}}{\partial N_A} & \frac{\partial \mathrm{t_R}}{\partial k} & \frac{\partial \mathrm{t_R}}{\partial b} \end{bmatrix} V \begin{bmatrix} \frac{\partial \mathrm{t_R}}{\partial N_A} \\ \frac{\partial \mathrm{t_R}}{\partial k} \\ \frac{\partial \mathrm{t_R}}{\partial b} \end{bmatrix}. \tag{16}$$

The P_{ts} is then calculated using asymptotic property of the MLE with lognormal distributions as [22]:

$$P_{\mathrm{ts}} = 1 - \Phi_{\log}\left(\Phi_{\log}^{-1}\left(C_{\mathrm{req}}; \ln\left(t_{req}\right), (1-s) \cdot \sqrt{\mathrm{Var}(t_R)}\right); \ln\left(t_{\mathrm{p}}\right), \sqrt{\mathrm{Var}(t_R)}\right) \tag{17}$$

With $\Phi_{\log}(x; \mu_{\log}, \sigma)$ and $\Phi_{\log}^{-1}(q; \mu_{\log}, \sigma)$ being the cumulative distribution function (cdf) and the quantile function (icdf) of the lognormal distribution respectively. Here, t_{p} corresponds to the lifetime for the load profile of interest according to the parameter set of prior knowledge under validity of H_1.

All the equations here are very similar to the ones already introduced in [13]. The concept and the hypothesis testing framework are the same. However, due to the expected Fisher-Information matrix, no failure times are needed, and the equations can directly be used for finding the best test settings for an expected test output.

3. Optimization: Finding the Best Accelerated Test Settings

For calculating the P_{ts}, eq. (17) can also be represented using normal distributions in cdf Φ and icdf form Φ^{-1}:

$$P_{\mathrm{ts}} = 1 - \Phi\left(\frac{\ln(t_{\mathrm{p}}) - \ln(t_{\mathrm{req}})}{\sqrt{\mathrm{Var}(t_R)}} - \Phi^{-1}(C)\right) \tag{18}$$

Since t_{p} is governed by prior knowledge and therefore fix for a single P_{ts}value and the reliability requirement is also fix, therefore t_{req}, R and C are fix as well. P_{ts} is hence only dependent on $\mathrm{Var}(t_R)$. It is evident that P_{ts} increases as $\mathrm{Var}(t_R)$ decreases. Since the load profile is known and fixed for evaluating reliability and lifetime (fixed values of $\nabla\Theta$), analyzing $\mathrm{Var}(t_R)$ via eq. (12)-(16) reveals it only depends on the total number of test specimen n and well as the load level height (and number). Since a third load level (or more) has proven to be not practically relevant in a reliability demonstration sense (see [14], [15], [17], [27]), the focus is on two load levels only here. The total number of test specimens is usually restricted by the budget available for testing, for this reason, it shall also be considered fixed in the following analyses. From eq. (12) it can already be seen that the entries of I increase with increasing n which in turn results in decreased values of V and therefore smaller $\mathrm{Var}(t_R)$. What remains is an optimization of load level height S_I, S_{II} and specimen allocation onto those levels n_I, n_{II}.

The optimization problem can therefore be formulated as

$$\text{minimize:}$$
$$f\left(\frac{n_I}{n}, S_I, S_{II}\right) = \mathrm{Var}(t_R) = \nabla\Theta^T V \nabla\Theta \tag{18}$$

This is aligns with the findings by Herzig [17], [27] in a sense that one has to focus on the estimated lifetime quantile and the variance of it due to estimation by the test rather than model estimation itself. Herzig, however, does not make use of the hypothesis testing approach but instead emulates the test evaluation procedure and thus is required to use two consecutive estimations of the lifetime quantile, which is solved by application of a doubled Monte-Carlo approach. Although being conceptually contrasting, it also aligns with the general specimen allocation by Nelson [7] who also makes some more simplified assumptions in his analyses. However, by establishing the equation as done here, the optimization can be performed in seconds rather than minutes or even hours, since no Monte-Carlo Simulation or Bootstrap needs to take place. It must be noted, however, that a solely analytical formulation is not possible for the optimum while considering all three parameters of the lifetime model (N_A, k, b).

4. Exemplary Calculation: Case Study

A reliability demonstration of a simple shaft of a traction motor shall be performed which is stressed by cyclic torque which results in torsional stress. The Wöhler-Weibull lifetime model is known from

previous products and shall therefore be used for the accelerated test planning. The values are $N_A = 1E6$, $S_A = 60$ MPa, $k = 5$ and $b = 2.5$. The load profile is simplified for this example to be $S_{\text{load}} = [180, 60, 30]$ MPa and $N_{\text{load}} = [3.28E2, 1.31E4, 1.64E6]$ cycles. Since the highest stress without changing failure modes is 220 MPa and the lowest load due to test infrastructure is 70 MPa, these values shall be used as maximum and minimum for the load level heights respectively. The available budget for testing shall be a maximum of $n = 30$ as the total sample size. So, the engineering challenge here is to find the two test load levels S_I and S_{II} with their respective sample size allocation so that P_{ts} is maximized and the chance of successfully demonstrating the reliability requirement of $R_{\text{req}} = C_{\text{req}} = 90$ % can be maximized as well.
To analyze the influence of sample size and lifetime requirement on the sample allocation, a fixed set of test load levels is calculated, see Fig. 1.

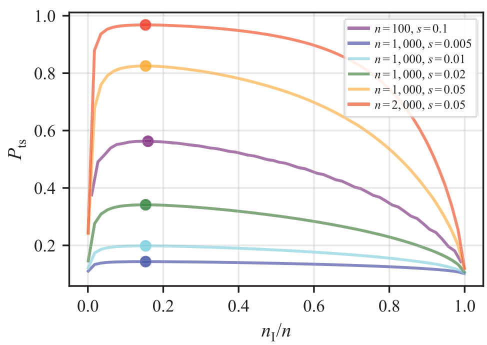

Fig. 1. Optimal allocation for fixed load levels with respect to total sample size and lifetime requirement (N_{load} is scaled linearly so that $s = 1 - D_{\text{req}}/D_{\text{load}}$).

The optimal allocation does not change with sample size and lifetime requirement N_{load}. However, changing the load levels, the optimal allocation changes, see Fig. 2.
A greater spread of the load levels benefits the P_{ts} and an optimal allocation calls for more specimen on the lower load level with increasing spread.
Using eq. (18) for the minimization problem and a simple minimizing algorithm (e.g. SciPy's minimize [28]), the problem can be solved in a straightforward manner.

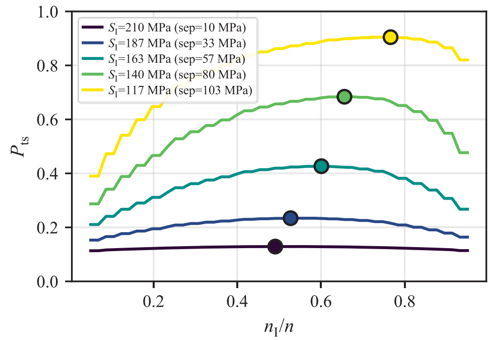

Fig. 2 Optimal sample allocation for varying lower load level and fixed upper load level at 220 MPa for total sample size of 30.

The result is to test $n_I = 24$ on the lower load level of $S_I = 70$ MPa and test $n_{II} = 6$ specimen on the upper load level of $S_{II} = 220$ MPa which was set to the maximum possible value in order to ensure a shorter test. The optimization algorithm yields a maximum spread of the load levels, which is in accordance with Fig. 2. A maximum of $P_{\text{ts}} = 99.54$ % can be reached. The optimal allocation allows for a good overview of the change of P_{ts} with different allocations, see Fig. 3.

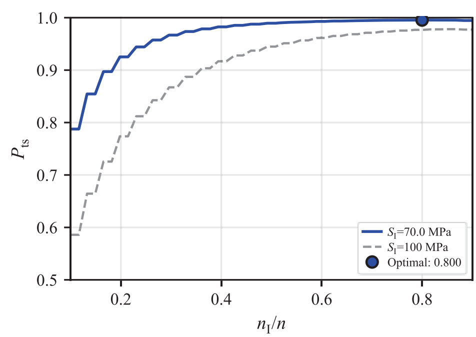

Fig. 3. Change of P_{ts} with change of allocation for maximum spread of load levels and total sample size of $n = 30$.

It is also evident how much the P_{ts} will drop when the lower load level is raised to 100 MPa.
Since the load profile being used for the demonstration should always also represent the field load profile shape [26], the effect of a block profile is further analyzed. The optimization for a block profile at 25 MPa is calculated to gather insight on the change of optimal test settings. The results are: $n_I = 21$, $n_{II} = 9$, $S_I = 70$ and again

$S_{II} = 220$ MPa fixed. This highlights a crucial aspect of reliability demonstration for accelerated test: One must always use representative load profile shapes for demonstration. Otherwise, the estimated lifetime will be influenced by test design and not be optimal or even not representative. Because to get a higher P_{ts} here due to the larger portion of extrapolation, the lower load level must have more failures.

This implies one is advised to use the actual load profile shape of field load, already in test planning. This insight in contrast to the widely used block profile approach with equivalent damage [24]

5. Summary and Conclusions

The concept of employing the Probability of Test Success is used here for accelerated reliability demonstration tests. The already established equations for the Wöhler-Weibull model in the context of hypothesis testing for reliability were further developed to use the expected Fisher-Information matrix. This allows for a clear view on the relevant influences for the optimization problem of finding the optimal test settings for a maximum in P_{ts}. It is shown that in contrast to common optimality criteria, the variance of the lifetime quantile itself must be in focus and is solely responsible for the achieved value of P_{ts} of the test, for the scenario of fixed reliability requirement and products performance (lifetime model parameters). A brief example shows the advantage of the methods and emphasizes the shape of the load profile to be considered already in the test planning. It is also seen, that a wider spread of the load levels as well as an allocation of samples onto the lower load level is beneficial to the P_{ts}. With the proposed optimization, the load level height as well as the allocation of the test specimen onto the levels can be determined automatically. Since the approach and subsequent equations are different from the approach presented in [16], but seem to coincide with regard to their motivation, further studies of comparing the two would be helpful. A further addition could also consider the actual time and specimen-depending cost of the test, which will further refine the results by weighing the influencing factors according to the individual company's capabilities and cost structure.

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