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An active-learning Kriging model for non-probabilistic reliability analysis

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Traditional reliability analysis methods are generally based on probability theory, where parameter uncertainties are represented as random variables. which is often impractical, especially in the early stages of product design when limited data are available. To address this issue, a non-probabilistic reliability analysis method has been developed. Among these, the Interval variables method provides major advantages in addressing insufficient information. However, for systems with implicit and computationally expensive limit-state functions, determining the non-probabilistic reliability index remains a major challenge. To resolve this challenge, the Efficient Global Optimization (EGO) approach offers a promising way. Constraints in non-probabilistic reliability analysis ensure that the optimization process yields solutions that are physically feasible and meet the practical requirements of engineering design. This paper proposes a new strategy to handle the constraints for non-probabilistic reliability analysis in the framework of Efficient Global Optimization. The inequality constraint is merged with the performance function, so both the constraint and the performance function are considered, ensuring that the resulting solutions satisfy practical engineering requirements. The Kriging model serves as a surrogate for the computationally demanding simulations, while the EGO process actively guides the search for optimal bounds of uncertainty parameters. The applicability and efficiency of the proposed method are verified through examples, demonstrating improved feasibility, reduced computational cost, and enhanced practicality in structural reliability analysis.

Keywords: Surrogate model, non-probabilistic reliability index, learning function, constraint, Kriging, efficient global optimization.

1. Introduction

In general, reliability analysis methods are based on probability theory, where parameter uncertainties are represented as random variables, which requires a large number of samples. Meeting this requirement proves challenging, particularly in the early stages of product design, making existing probabilistic-based reliability analysis methods face difficulties. To address this issue, a non-probabilistic reliability analysis method is explored in this paper. Interval variables, which is non-probabilistic reliability analysis method, provide major advantages in addressing insufficient information or samples since they require just a small amount of data to obtain lower and upper bounds of uncertainty parameters. In interval-based non-probabilistic reliability analysis, the primary goal is to calculate the system's non-probabilistic reliability index. When facing the time-consuming implicit functions of the limit states, calculating the system's non-probabilistic reliability index becomes challenging. To resolve this challenge, the efficient global optimization (EGO) approach offers a promising way. (Jones et al. 1998)

EGO leverages the Kriging model, a statistical method that predicts the behavior of complex functions based on limited observed samples. By utilizing the Kriging model, EGO can efficiently explore the solution space to identify the global minimum and maximum values of the performance function $g_i(Y)$ EGO also minimizes the computational burden associated with evaluating the time-consuming implicit functions, making it suitable for complex engineering systems with limited computational resources. (Zaefferer et al. 2014)

Additionally, by iteratively refining the Kriging model and strategically selecting new training samples, EGO effectively balances exploration and exploitation to converge to the extremal values of $g_i(Y)$. Overall, the integration of EGO with the Kriging model offers a robust and efficient approach to handling time-consuming implicit functions in optimization problems. (Viana et al. 2013)

Considering constraints on the performance function plays a pivotal role in ensuring that the obtained solutions are aligned with practical requirements. These constraints, which may encompass physical laws, design specifications, or safety standards, serve to mirror the constraints that the system must adhere to in actual operational environments. By incorporating such constraints, non-probabilistic analysis can identify solutions that not only meet performance objectives but also satisfy relevant practical limitations.

Genetic Algorithm (GA) are widely used for solving optimization problems across various fields due to their ability to efficiently explore complex solution spaces and find near-optimal solutions even in the presence of constraints. Genetic Algorithms efficiently explore the solution space, considering both the objective function and constraints to identify the optima. (Wang et al. 2021)

A strategy is proposed to handle constraints in this paper, and combined with the EGO method to optimize the extreme values, which is effective compared with the GA. Through the comparative analysis of the two examples, the efficiency of the proposed method is measured by solution quality, convergence speed, and the number of performance function calls.

2. Non-probabilistic Reliability Index Model

The non-probabilistic index method is an alternative to assess the reliability or safety of engineering systems compared with probabilistic models. Unlike probabilistic methods, which incorporate uncertainties in system parameters through probability distributions, the non-probabilistic index method evaluates system reliability based on the maximum and minimum of the system performance. (Jiang et al. 2014)

Several studies have corroborated the efficacy of the non-probabilistic index η as a suitable metric for assessing the safety level of structures. By leveraging the maximum and minimum of the system performance, the index η offers a straightforward yet effective approach to evaluating the structural integrity and failure probability. Its simplicity and direct applicability make it particularly appealing in scenarios where distribution functions of random variables may be limited or impractical to obtain. Its endorsement by multiple studies underscores its significance as a reliable metric for assessing the state of structures and ensuring their resilience under operational conditions. The formulation of the non-probabilistic index η is shown in Eq.(1).

$$\eta = \frac{y_{\max}+y_{\min}}{y_{\max}-y_{\min}} \qquad (1)$$

where $Y_{\min}$ and $Y_{\max}$ are the minimum and maximum values obtained by the proposed method. A higher value of the reliability index indicates a more reliable or safer system, while a lower value implies a higher risk of failure.

3.Proposed Method

Integrating an active learning Kriging model into non-probabilistic reliability analysis, and incorporating inequality constraints with the performance function gives a great advantage in computing the non-probabilistic index.

This paper proposed a strategy to handle constraints and compute the non-probabilistic reliability index of a system. The traditional method of time-consuming simulations is replaced with a Kriging model, which is further enhanced through active learning. Through EGO, new training samples are identified to refine the Kriging model and continue to optimize. Additionally, the expected improvement (EI) learning function is utilized to iteratively identify the optimal value throughout the process. By adopting this approach, the reliability analysis becomes more streamlined and computationally efficient, paving the way for a system non-probabilistic reliability index. (Zhan and Xing 2020).

The evaluation of the minimum and maximum bounds of the system performance during the design stage may require a large number of simulations of the original performance function. For complex engineering systems, these simulations can be computationally expensive and time-consuming, which makes direct application of optimization methods impractical. To address this issue, a Kriging-based metamodel is employed to approximate the original performance function. The Kriging surrogate significantly reduces the computational burden by replacing repeated evaluations of the expensive model with a fast analytical approximation, while maintaining acceptable prediction accuracy. This strategy allows the Efficient Global Optimization (EGO) framework to efficiently search for the minimum and maximum bounds of the system performance, making the proposed approach more suitable for practical engineering applications where computational cost is a critical concern.

In essence, considering constraints in non-probabilistic analysis must consider both the performance goals and practical considerations, ultimately leading to more effective and reliable system designs. The constraint in this paper is an inequality constrain which is given in Eq.(2).

$$x_1 - x_2 \leq 0 \qquad (2)$$

where the constrain represents an inequality constraint between two variables, x_1 and x_2. In practical terms, it means that the value assigned to x_1 must be less than or equal to the value assigned to x_2 . This constraint imposes a specific relationship between the two variables that must be satisfied throughout the optimization process.
The process described entails three key stages. Initially, the method involves constructing an initial Kriging model based on a limited set of training samples. Kriging modeling provides a means to predict complex function behaviors utilizing a small size of samples. By constructing this initial Kriging model, the algorithm gains a preliminary understanding of the underlying function's behavior, offering an initial optimum for exploration within the solution space. Despite its reliance on a small number of training samples, the initial Kriging model serves as a foundation upon which subsequent iterations of optimization can proceed.
Following the construction of the initial Kriging model, the algorithm employs the EGO method. EGO iteratively enhances the Kriging model's accuracy by strategically selecting and incorporating additional training samples at each iteration with consideration of the constraints. Through a balanced approach of exploration and exploitation, EGO aims to systematically explore promising regions of the solution space while refining the optima's accuracy. This iterative refinement process guides the optimization process towards the global minimum and maximum values of the objective function and ensures that the optima is more accurate. (Viana et al. 2013)
The optimization process continues iteratively until a predefined stopping criterion is met. Upon reaching termination, the final optima of the performance function are accurate. Leveraging EGO, the algorithm finds the global minimum or maximum values of the objective function in consideration of the constraint, providing practical and reasonably optimal solutions.
The proposed method considers the performance function with inequality constraints, indicating that the optimization process considers both the objective function and limitations imposed by inequality constraints.

3.1.*Learning Function*

The application of EGO improves optima-finding through enhanced exploration-exploitation trade-offs of known promising areas. This strategic selection of the points maximizes the algorithm's ability to efficiently converge toward optimal solutions while minimizing the computational burden. To minimize the objective function, the $\mathrm{EI}_{\min}(\mathbf{x})$ learning function is shown in Eq.(3).

$$\mathrm{EI}_{\min}(\mathbf{x}) = \left(\mathrm{y}_{\min} - \mathrm{m_n}(\mathbf{x})\right)\Phi\left(\frac{\mathrm{y}_{\min}-\mathrm{m_n}(\mathbf{x})}{\sigma_\mathrm{n}(\mathbf{x})}\right) \quad (3)$$
$$+\sigma_\mathrm{n}(\mathbf{x})\phi\left(\frac{\mathrm{y}_{\min} - \mathrm{m_n}(\mathbf{x})}{\sigma_\mathrm{n}(\mathbf{x})}\right)$$

For maximizing the objective function, the $\mathrm{EI}_{\max}(\mathbf{x})$ learning function is shown in Eq.(4).

$$\mathrm{EI}_{\max}(\mathbf{x}) = \left(\mathrm{m_n}(\mathbf{x}) - \mathrm{y}_{\max}\right)\Phi\left(\frac{\mathrm{m_n}(\mathbf{x})-\mathrm{y}_{\max}}{\sigma_\mathrm{n}(\mathbf{x})}\right) \quad (4)$$
$$+\sigma_\mathrm{n}(\mathbf{x})\phi\left(\frac{\mathrm{m_n}(\mathbf{x}) - \mathrm{y}_{\max}}{\sigma_\mathrm{n}(\mathbf{x})}\right)$$

The expressions of EI learning function for minimizing and maximizing objectives in optimization problems utilizing surrogate models like Kriging model offer a means to guide the selection of new samples for evaluation. By maximizing this expected improvement, algorithms like EGO efficiently explore the solution space to converge toward optimal solutions, The new sample is added with y^*_{min} and y^*_{max} can be obtained as shown in Eq.(5).

$$\begin{cases} y^*_{min} = \text{argmax}[\text{EI}_{min}(y)] \\ y^*_{max} = \text{argmax}[\text{EI}_{max}(y)] \end{cases} \quad (5)$$

Represent the values of y that maximize the expected improvement for minimizing and maximizing objectives, respectively. These values are determined by identifying the points where the expected improvement is maximized, based on the surrogate model's predictive mean and standard deviation. By finding the optimized y^*_{min} or y^*_{max}.

3.2. *Proposed Strategy for Handling the Constraints*

The extended EI learning function represents the expected improvement for a given point x in an optimization problem, subject to a constraint $g(\mathbf{x}) \leq 0$. Specifically, if the constraint $g(\mathbf{x})$ is satisfied the expected improvement remains unchanged and is denoted as $EI(\mathbf{x})$. However, if the constraint is violated, $g(\mathbf{x}) > 0$, indicating that the point x is infeasible, the expected improvement is set to zero. This formulation ensures that only feasible points, which contribute to the expected improvement and satisfy the constraints, are searched, and the extended EI learning function is shown in Eq.(6).

$$\text{EI}(\mathbf{x}) = \begin{cases} \text{EI}(\mathbf{x}), \text{if } g(\mathbf{x}) \leq 0 \\ 0, \ \text{if } g(\mathbf{x}) > 0 \end{cases} \quad (6)$$

In this paper, $g(\mathbf{x}) = x_1 - x_2$ and the optimization process begins by initializing a set of initial training samples using Latin Hypercube Sampling (LHS). The surrogate model is then constructed based on these training samples, followed by the calculation of the maximum or minimum objective function value from the initial samples. Within each iteration, the extended EI criterion is employed to identify the next search point for evaluation. Once the candidate point with the highest EI value is identified, it is evaluated using the true objective function, and the sample set and the optima are updated accordingly. The process continues iteratively until the maximum and minimum number of evaluations is reached or until a stopping criterion is met.

3.3. *Stopping Criterion*

The stopping criterion utilized in the proposed method serves as an indication to stop the iterative optimization process once the expected improvement is sufficiently small. This threshold represents an extremely small value, indicating that further improvements in the objective function are negligible compared to the current optimal, thereby preventing unnecessary computational overhead and resource consumption, and the stopping criterion is shown in Eq.(7).

$$\max\{\text{EI}(\mathbf{x})\} \leq 10^{-5} \quad (7)$$

By incorporating this stopping criterion, the optimization algorithm intelligently terminates once the expected improvement diminishes to a level deemed insignificant.

4. Steps of the proposed method

The proposed method aims to integrate an active learning Kriging model into non-probabilistic reliability analysis by incorporating inequality constraints with the performance function. The proposed method can be employed in seven main steps, the flowchart is shown in Figure 1, each of which is briefly described below:

Step 1: Generate initial random samples within the specified design space using Latin Hypercube Sampling.

Step 2: Construct the initial Kriging model using the generated samples and corresponding function evaluations.

Step 3: Determine the candidate point with the highest extended EI value by using the GA optimization approach

Step 4: Verify whether the stopping criterion has been met. If it has, proceed to Step 8 in the active-learning process. If not, proceed to Step 6.

Step 5: If the stopping condition at Stage 6 is not met, the optimization process continues, and the best search point is incorporated into the design of experiments. Subsequently, this best point is assessed using the actual performance function to further inform the optimization process.

Step 6: Calculate the y_{min} or y_{max}

Step 7: Calculate the non-probability reliability index using the min and max value

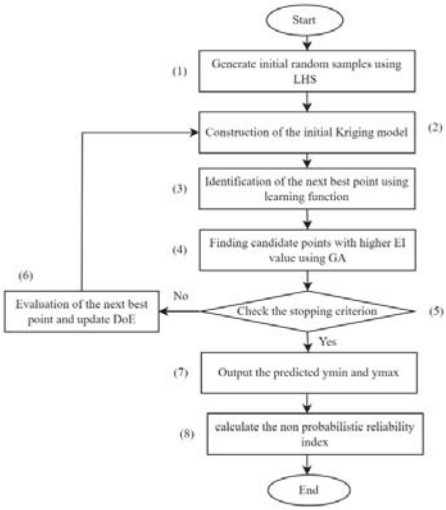

Fig. 1. Flow chart of the proposed active-learning Kriging model method for the non-probabilistic reliability index.

5. Examples, Results, and Discussion

The applicability of the proposed method is demonstrated through the investigation of two numerical examples. The first example involves a two-dimensional test function, while the second example concerns a parallel system. For each example, the non-probabilistic reliability index of the system calculated using the GA is reported. These examples serve to illustrate the effectiveness and practicality of the proposed method.

5.1. *Examples and Results*

Example 1 A two-dimensional test function

The two-dimensional test function is highly nonlinear is shown in Eq.(8). (Fu et al. 2018)

$$y = (1.5x_1 - 2)^2 - (x_2 - 3)^2 + x_1x_2 + 10\sin(2\pi x_1) + 10\sin(2\pi x_2) \quad (8)$$

The constraint of this example is $g(\mathbf{x}) = x_1 - x_2$. Table 1 demonstrates that the proposed method effectively reduces the number of iterations required for finding the minimum and maximum values compared with the GA, without compromising the accuracy of the non-probabilistic reliability index.

Table 1. Non-probabilistic reliability index results of a two-dimensional test function.

Methods	GA	The proposed method
Number of iterations for min value	3650	106
Number of iterations for max value	3400	123
Total number of iterations	7050	229
Min $g(x)$	-23.75	-23.48
Max $g(x)$	38.91	38.89
η	0.2418	0.2471

In terms of accuracy, the result obtained from the proposed method for the non-probabilistic reliability index is 0.2071, which is close to the benchmark result of 0.2418, and the relative error is small, thus, less than 5% is acceptable. In terms of efficiency, the proposed method only requires 106 iterations while GA requires 3650 iterations to get the minimum value and for the maximum value, the proposed method requires only 123 iterations while GA requires 3400 iterations, which the proposed method significantly reduces the function call from several thousand to hundreds, and save the computational cost.

Figure 2 and Figure 3 show the procedure for identifying the minimum value and maximum value, respectively using the proposed method.

In figure 2 the initial value is -16.8, after 11 iterations it gets the value of -19.2 but is not the global minimum after more six iterations the min is reduced and gets the global min and nearly only 30 iterations is required to get the global min. For the global max it also needs a small number of iterations. Overall, the sequential process of finding the minimum value by the proposed method involves a systematic and iterative approach to refine the search space and converge toward the optimal solution while ensuring accuracy and efficiency in the estimation of the non-probabilistic reliability index.

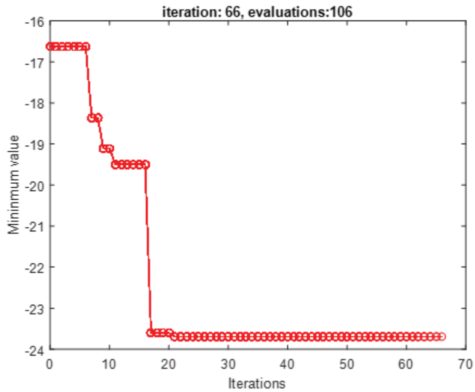

Fig. 2. The procedure for identifying the minimum value using the proposed method.

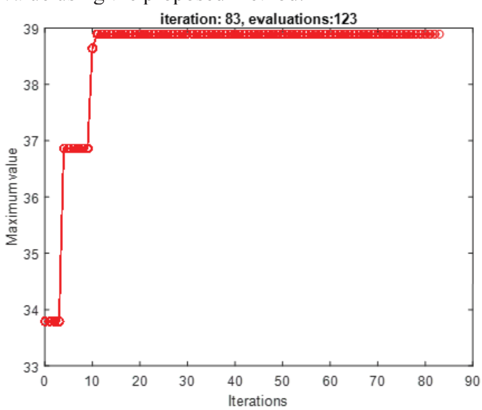

Fig. 3. The procedure for identifying the maximum value using the proposed method.

Example 2 Parallel system function

The parallel system has two performance function α is equal to 4 as shown in Eq.(9). (Grooteman 2008)

$$\begin{cases} g_1(Y_1, Y_2) = 2 - Y_2 + exp(-0.1Y_1^2) + (0.2Y_1)^4 \\ g_2(Y_1, Y_2) = \alpha - Y_1Y_2 \end{cases} \quad (9)$$

The constraint of this example is $g(\mathbf{x}) = x_1 - x_2$. Table 5-2 demonstrates the proposed method efficiently decreases the iteration count needed to locate both the minimum and maximum values, all while maintaining the accuracy of the result, non-probabilistic reliability index. In terms of accuracy, the result obtained from the proposed method for the non-probabilistic reliability index is 0.4625 which is the same as the bench mark result of 0.4625.

Table 2. A parallel system function non-probabilistic reliability index.

Methods	GA	The proposed method
Number of iterations for min value	3650	106
Number of iterations for max value	3400	123
Total number of iterations	7050	229
Min $g(x)$	-23.75	-23.48
Max $g(x)$	38.91	38.89
η	0.2418	0.2471

In terms of efficiency, the proposed method only requires 43 total iterations (22 iterations to get the minimum value and 21 iterations to get the maximum value) while GA requires 5400 total iterations (2800 iterations to get the minimum value and 2600 iterations to get the maximum value), which shows the proposed method significantly reduces the function call, which saves the computational cost.

Figure 5-2 and Figure 5-3 show the procedure for identifying the minimum value and maximum value, respectively using the proposed method. The initial value is -0.95, after 10 iterations it gets the global min. For the global max it also needs a small number of iterations.

This sequential process ensures a systematic and iterative refinement of the search space, ultimately converging towards the optimal solution. Throughout this process, emphasis is placed on maintaining both accuracy and efficiency in estimating the non-probabilistic index. Overall, the method's approach combines systematic iteration with careful adjustments to refine the search space effectively and achieve reliable results.

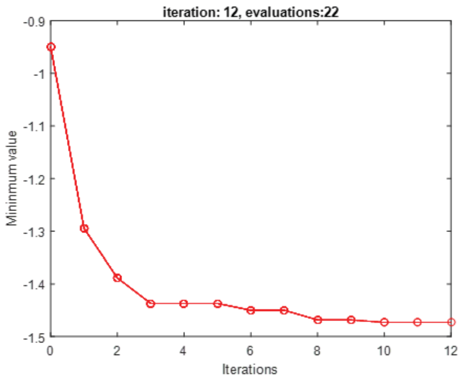

Fig. 4. The procedure for identifying the minimum value using the proposed method involves a sequential approach.

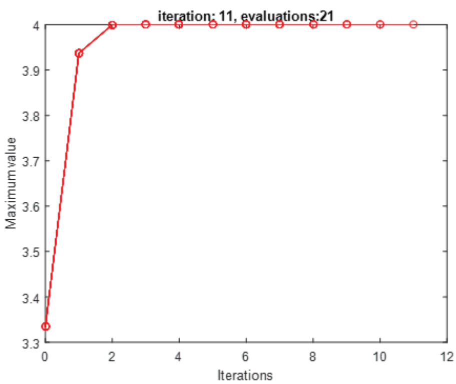

Fig. 5. The procedure for identifying the maximum value using the proposed method involves a sequential approach.

6. Summary

Integrating an active learning Kriging model into non-probabilistic reliability analysis, along with incorporating inequality constraints within the performance function, offers a significant advantage in non-probabilistic reliability index computations. This approach jointly considers both constraints and the performance function, ensuring solutions align with practical requirements such as safety standards and design specifications. By leveraging efficient global optimization and iterative refinement through active learning, the traditional reliance on time-consuming simulations is replaced with a streamlined and computationally efficient process. The proposed approach ensures that solutions are not only optimal but also practical and feasible, enhancing the accuracy and applicability of system assessments in practical scenarios.

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