Proc. of the European Safety and Reliability Conference (ESREL 2026)
Edited by José C. Matos, Paulo B. Lourenço, Michael Beer, Daniel Oliveira,
Edoardo Patelli, Hélder S. Sousa, Rui A. Silva and Monica Santamaria

doi: 10.3850/ESREL2026061419_esrel26-p26179-cd

A Risk Assessment Model of Complex Systems Based on Evidential Reasoning Rule with Dependent Evidence

Tang Shuaiwen

College of Systems Engineering, National University of Defense Technology, China, E-mail: 631845201@qq.com

Jiang Ping

College of Systems Engineering, National University of Defense Technology, China, E-mail: jiangping@nudt.edu.cn

Yu Haiyue

College of Systems Engineering, National University of Defense Technology, China, E-mail: haiyue_nudt@163.com

Abstracts: In this paper, an innovative model called evidential reasoning rule with de-pendent evidence (ERR-DE) is proposed for the risk assessment of complex systems under uncertainties. A general risk assessment indicator system is established, and the risk indicators are modelled and described as evidence under the discernment of framework (FoD). The evidence is unified by the transformation matrix, and the evidence reliability and dependence index of evidence are explicitly measured. The ERR-DE model forms a multi-information fusion framework, where multiple pieces of evidence with different weights, reliabilities and dependence indexes are aggregated to establish the relationship between risk indicators and system risk. A parameter optimization model is constructed, where all subjective evidential parameters can be learned through the idea of maximizing expectations. The proposed model is used to assess the risk of a gyroscope.

Keywords: Risk assessment, inferential modelling, evidential reasoning rule, uncertainty, evidence dependence.

1. Introduction

Risk assessment aims to quantitatively assess the possible degree of impact or loss caused by a risk event before or after the event, which is of great significance (Siami et al. 2024, Zhou et al. 2025). Existing risk assessment methods can be divided into three categories. The knowledge-based method establishes a systematic assessment model using mechanism and domain knowledge, such as the analytic hierarchy process, Petri net, fault tree, etc. However, when the system mechanism is complex, it is difficult to obtain the domain knowledge. The data-driven method does not require precise acquisition of the analytical model, including artificial neural network, deep learning, etc. However, it relies on large and accurate samples, lacking interpretability. In comparison, the hybrid information-based method can combine qualitative knowledge and quantitative data to effectively assess the risk degree of systems using small samples, including Bayesian network, fuzzy comprehensive evaluation, belief rule base, evidential reasoning rule (ERR). This method provides a sound idea for the risk assessment of complex systems.

As a typical hybrid information-based method, ERR forms a generalized probabilistic reasoning scheme to achieve multi-source information fusion, which extends traditional Bayes' rule and Dempster's rule (Yang and Xu 2013). It constructs the framework of discernment (FoD) to model uncertainties. By using ERR, the system inputs can be transformed to evidence with weight and reliability, which can be aggregated to build a bridge between system inputs and output. So far, ERR has been widely applied in the risk assessment of complex systems (Fu et al. 2023, Gizem et al. 2023). In ERR, however, the FoD of evidence should strictly correspond to that of reasoning results. This means the number of referential values of input indicators is consistent with the number of assessment grades of output. It is difficult since different experts can provide different types and quantities of assessment grades according to the system design principle and standard. Besides, ERR requires all the evidence to be independent of each other, which seems idealized due to the coupling of system structure. Although the maximum likelihood evidential reasoning (MAKER) framework has been proposed to aggregate interdependent evidence, it requires joint probabilities and marginal probabilities under all combinations, resulting in exponential computational complexity (Yang and Xu 2025, Zhang et al. 2023). Most importantly, with the increase of the number of risk indicators, experts will find themselves in a dilemma to give all evidential parameters due to limited knowledge.

Based on the above analysis, the paper aims to introduce a risk assessment model for complex systems, referred to as evidential reasoning rule with dependent evidence (ERR-DE). The contributions are as follows. Firstly, a transformation matrix-based evidence unification method is proposed, which aligns the assessment framework of input risk indicators with output results. Secondly, to better express the subjective judgment and personal preferences of experts, the evidence weight is described in interval form. A calculation method of evidence reliability is introduced based on static and dynamic reliability. Thirdly, a calculation method of dependence index between evidence is proposed, and the relative total dependence coefficient (RTDC) is defined by the distance correlation. Finally, to alleviate the subjective uncertainty brought by experts, a parameter optimization model is constructed, which can improve the risk assessment accuracy while keeping the interpretability.

2. Proposed Risk Assessment Model

Suppose there are L risk indicators denoted by $x_1, x_2, \cdots, x_L$, which can be monitored by L different sensors. A general risk assessment indicator system can be constructed as Fig. 1.

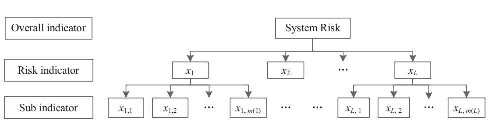

Fig. 1. General risk assessment indicator system.

In Fig. 1, the number of sub indicators as $m(l)$ can be refined according to the actual situation, and $l \in [1, L]$. Suppose the observation data of x_i at time instant t is $x_i(t)$, which can provide a piece of evidence at t, denoted by $e_i(t)$. Correspondingly, the evidence weight and reliability can be denoted by w_i and $r_i(t)$ respectively, $0 \le w_i, r_i(t) \le 1$. The evidence reliability is time-varying, consisting of static reliability r_i^s and dynamic reliability $r_i^d(t)$. The dependence index between $e_i(t)$ and all evidence is $d_i(t)$.

2.1.Evidence Acquisition Method and Unification Method

Suppose the FoD of risk indicator x_i is $\Theta_i = \{H_{i,1}, H_{i,2}, \cdots, H_{i,n(i)}\}$, where $H_{i,k}$ is the kth risk assessment grade of x_i, and $i \in [1, L]$ and $k \in [1, n(i)]$. Here, $n(i)$ refers to the number of grades of x_i. The referential value of $H_{i,k}$ is assumed as $h_{i,k}$. If x_i is a benefit indicator, there is $h_{i,1} \le h_{i,2} \le \cdots \le h_{i,n(i)}$. If x_i is a cost indicator, there is $h_{i,1} \ge h_{i,2} \ge \cdots \ge h_{i,n(i)}$. Without loss of generality, suppose $h_{i,k} \le x_i(t) \le h_{i,k+1}$, where $k, k+1 \in [1, n(i)]$. The utility-based equivalence transformation technique can be used to acquire evidence (Yang 2001), shown as follows:

$$\beta_{i,k}(t) = \frac{h_{i,k+1} - x_i(t)}{h_{i,k+1} - h_{i,k}} \quad (1)$$

$$\beta_{i,k+1}(t) = 1 - \beta_{i,k}(t) \quad (2)$$

$$\beta_{i,j}(t) = 0; j \ne k, k+1 \text{ and } j \in [1, n(i)]. \quad (3)$$

where $\beta_{i,k}(t)$, $\beta_{i,k+1}(t)$ and $\beta_{i,j}(t)$ are belief degrees that $x_i(t)$ is assessed to grades $H_{i,k}$, $H_{i,k+1}$ and $H_{i,j}$ respectively.

Based on the above analysis, a piece of evidence can be generated from $x_i(t)$, shown as the following belief distribution:

$$e_i(t) = \left\{\left(H_{i,j}, \beta_{i,j}(t)\right); j \in [1, n(i)]\right\} \quad (4)$$

Suppose the FoD of risk assessment result is $\Theta = \{H_1, H_2, \cdots, H_n\}$, and a piece of evidence under Θ can be profiled by:

$$e(t) = \left\{\left(H_p, \beta_p(t)\right); p \in [1, n]\right\} \quad (5)$$

where $\beta_p(t)$ is the belief degree to grade H_p.

As mentioned in Section 1, the FoD of evidence may differ from that of result, which means $n(i)$ is not necessarily euqal to n. Thus, (4) and (5) are different. To unify different FoDs, the concept of transformation matrix is introduced to establish the mapping relationship between Θ_i and Θ, shown as follows:

$$\mathbf{A}_i = \begin{bmatrix} \alpha_{1,1} & \alpha_{2,1} & \cdots & \alpha_{n,1} \\ \alpha_{1,2} & \alpha_{2,2} & \cdots & \alpha_{n,2} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{1,n(i)} & \alpha_{2,n(i)} & \ldots & \alpha_{n,n(i)} \end{bmatrix} \quad (6)$$

where $\mathbf{A}_i$ is the transformation matrix with a dimension of $n(i) \times n$. Each row of $\mathbf{A}_i$ can be denoted by an "IF-THEN" rule as $R_{i,k}$. $R_{i,k}$ means that "IF x_i is $H_{i,k}$, THEN $\{(H_1, \alpha_{1,k}), (H_2, \alpha_{2,k}), \cdots, (H_n, \alpha_{n,k})\}$, with $0 \le \alpha_{p,k} \le 1$ and $\sum_{p=1}^{n} \alpha_{p,k} \le 1$".

Suppose the belief degree matrix of e_i is $\boldsymbol{\beta}_i = [\beta_{i,1}, \beta_{i,2}, \cdots, \beta_{i,n(i)}]$, and the belief degree matrix of e is $\mathbf{B} = [\beta_1, \beta_2, \cdots, \beta_n]$. According to (6), we have

$$\mathbf{B} = \boldsymbol{\beta}_i \cdot \mathbf{A}_i \quad (7)$$

As such, all evidence can be unified to the same FoD as Θ in (5).

2.2. Calculation Methods of Evidence Weight and Reliability

To effectively deal with subjective uncertainty and fully leverage the role of experts in empowerment, the interval is employed to describe the evidence weight. Suppose there are M experts $E_1, E_2, \cdots, E_M$ assigning different evidence weights as $w_{i,1}, w_{i,2}, \cdots, w_{i,M}$ to x_i. All weights can be arranged in ascending order as $w_{i,1} \le w_{i,2} \le \cdots \le w_{i,M-1} \le w_{i,M}$. The interval weight of x_i can be given by $w_i \in [w_{i,2}, w_{i,M-1}]$ with the maximum and minimum weights deleted.

In the context of sensor, the evidence reliability is composed of static reliability and dynamic reliability (Tang et al. 2022). The static reliability r_i^s is an objective attribute of sensors, which is a fixed value. The dynamic reliability $r_i^d(t)$ is a subjective attribute of sensors (Zhao et al. 2018). Thus, the evidence reliability can be calculated as follows:

$$r_i(t) = \alpha r_i^s + (1-\alpha) r_i^d(t) \quad (8)$$

where $\alpha \in [0,1]$ is a discounting factor.

2.3. Calculation Method of Dependence Index

To make full use of reliable risk indicators, the evidence reliability can be used to determine the aggregation sequence (Yager 2009, Zhang et al. 2023). This indicates that the higher the reliability of a risk indicator, the higher its aggregation priority. In this paper, the dependence index of evidence is defined as RTDC using the distance correlation method.

For two pieces of evidence $e_i(t)$ and $e_j(t)$ at time instant t, the evidence reliabilities are $r_i(t)$ and $r_j(t)$ respectively. The relative aggregation sequence $Seq_{i,j}(t)$ can be determined by:

$$Seq_{i,j}(t) = \begin{cases} Seq_i(t) < Seq_j(t), & if\ r_i(t) > r_j(t) \\ Seq_{i,j}(t-1), & if\ r_i(t) = r_j(t) \\ Seq_i(t) > Seq_j(t), & otherwise \end{cases} \quad (9)$$

where $Seq_i(t)$ and $Seq_j(t)$ refer to the aggregation sequence of $e_i(t)$ and $e_j(t)$, respectively.

Suppose the observation data matrix of x_i at time instant t within interval length T is $\mathbf{Z}_i(t) = [x_i(t-T) \quad x_i(t-T+1) \quad \cdots \quad x_i(t)]$. Given two observation data matrices $\mathbf{Z}_i(t)$ and $\mathbf{Z}_j(t)$, the distance correlation coefficient of $e_i(t)$ and $e_j(t)$ can be calculated by (Székely et al. 2007):

$$C\left(\mathbf{Z}_i(t), \mathbf{Z}_j(t)\right) = \begin{cases} \frac{v^2(\mathbf{Z}_i(t), \mathbf{Z}_j(t))}{\sqrt{v^2(\mathbf{Z}_i(t)) v^2(\mathbf{Z}_j(t))}}, & if\ v^2(\mathbf{Z}_i(t)) v^2\left(\mathbf{Z}_j(t)\right) > 0 \\ 0, & if\ v^2(\mathbf{Z}_i(t)) v^2\left(\mathbf{Z}_j(t)\right) = 0 \end{cases} \quad (10)$$

Without loss of generality, if the aggregation sequence of $e_1(t), e_2(t), \cdots, e_L(t)$ is $\{1,2,\cdots,L\}$, the correlation matrix $\mathbf{C}$ is:

$$\mathbf{C} = \begin{bmatrix} C(\mathbf{Z}_1(t), \mathbf{Z}_1(t)) & 0 & \cdots & 0 \\ C(\mathbf{Z}_2(t), \mathbf{Z}_1(t)) & C(\mathbf{Z}_2(t), \mathbf{Z}_2(t)) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C(\mathbf{Z}_L(t), \mathbf{Z}_1(t)) & C(\mathbf{Z}_L(t), \mathbf{Z}_2(t)) & \ldots & C(\mathbf{Z}_L(t), \mathbf{Z}_L(t)) \end{bmatrix} \quad (11)$$

The total dependence degree of $e_i(t)$ can be calculated as follows:

$$Tdd_i(t) = \sum_{j=1}^{i} C\left(\mathbf{Z}_i(t), \mathbf{Z}_j(t)\right) \quad (12)$$

The dependence index between $e_i(t)$ and other evidence is $d_i(t)$, i.e., the RTDC value, which can be calculated by:

$$\overline{d_i}(t) = \frac{1/Tdd_i(t)}{\sum_{j=1}^{L} 1/Tdd_j(t)} \quad (13)$$

$$d_i(t) = \frac{\overline{d_i}(t)}{\max\{\overline{d_i}(t)\}}, i \in [1, L] \quad (14)$$

In the above equations, the smaller $d_i(t)$ is, $e_i(t)$ is more dependent on other evidence. When $d_i(t) = 1$, $e_i(t)$ is regarded as totally independent to other evidence.

2.4. ERR-DE-Based Risk Assessment Model

For a piece of evidence $e_i(t)$ profiled by (5) with weight w_i and reliability $r_i(t)$, in the FoD as $\Theta = \{H_1, H_2, \cdots, H_n\}$, the hybrid probability mass is calculated by:

$$m_{i,H}(t) = \begin{cases} 0, & H = \emptyset \\ \overline{w_i} d_i(t) \beta_{i,H}(t), & H \subseteq \Theta\ and\ H \ne \emptyset \\ 1 - \overline{w_i} d_i(t), & H = P(\Theta) \end{cases} \quad (15)$$

where $\emptyset$ denotes the empty set. $\overline{w_i}$ is the hybrid weight, and $\overline{w_i} = w_i/(1 + w_i - r_i(t))$. $P(\Theta)$ is the power set of Θ, composed of 2^n subsets of Θ. It can describe local and global ignorance in the risk assessment, profiled by:

$$P(\Theta) = \{\emptyset, \{H_1\}, \cdots, \{H_n\}, \{H_1, H_2\}, \cdots, \{H_1, H_2, \cdots, H_{n-1}\}, \Theta\} \quad (16)$$

For two pieces of evidence $e_i(t)$ and $e_j(t)$ profiled by (4), the evidence combination based on ERR-DE can be expressed as:

$$\beta_{e(2),H}(t) = \begin{cases} 0, & H = \emptyset \\ \frac{m_{e(2),H}(t)}{\sum_{\varphi \subseteq \Theta} m_{e(2),\varphi}(t)}, & H \subseteq \Theta \text{ and } H \neq \emptyset \end{cases} \quad (17)$$

$$m_{e(2),H}(t) = \sum_{\phi \cap \psi = H} m_{i,\phi}(t) m_{j,\psi}(t);\ \phi, \psi \subseteq \Theta \quad (18)$$

where $\beta_{e(2),H}(t)$ is the combined belied degree of H from $e_i(t)$ and $e_j(t)$. $m_{e(2),H}(t)$ is the unnormalized combined probability mass of H. $m_{i,\phi}(t)$ and $m_{j,\psi}(t)$ are hybrid probability masses of $e_i(t)$ and $e_j(t)$ assigned to risk grades ϕ and ψ respectively.

For L pieces of evidence, (17) and (18) can be recursively used $(L - 1)$ times to generate the risk assessment result, shown as follows:

$$S_{e(L)}(t) = \left\{\left(H, \beta_{e(L),H}(t)\right); H \subseteq \Theta\right\} \quad (19)$$

where $\beta_{e(L),H}(t)$ is the final combined belied degree of risk grade H. There is local and global ignorance in $S_{e(L)}(t)$ since H is not just a single subset. Thus, it is necessary to map $S_{e(L)}(t)$ to real probabilities of single subsets H_p for further assessment and decision (He et al. 2023). The calculation method is:

$$P_{H_p}(t) = \sum_{\theta \subseteq \Theta} \beta_{e(L),H}(t) \frac{|H_p \cap H|}{|H|};\ p \in [1, n] \quad (20)$$

where $P_{H_p}(t)$ is the risk probability that the system is at the risk grade H_p at time instant t.

Suppose the utility of H_p is $u(H_p)$, the expected utility of the system is:

$$RD(t) = \sum_{p=1}^{n} P_{H_p}(t) u(H_p) \quad (21)$$

where $RD(t) \in [0,1]$ can quantitatively measure the risk degree of system. A larger $RD(t)$ closer to 1 reflects a higher risk.

2.5. Proposed Parameter Optimization Model

In the ERR-DE model, several parameters are given by experts, including evidence weights, referential values of risk grades in different FoDs, and the transformation matrix. This will cause subjective uncertainty in the risk assessment result. As mentioned in Section 2.2, the evidence weight belongs to an interval $[w_{i,2}, w_{i,M-1}]$, which will make $RD(t)$ fluctuate within a certain range. Suppose the expected risk degree is RD, and the observed result is $RD(t)$. To alleviate the subjectivity of evidential parameters, a parameter optimization model can be established by maximizing the likelihood of expected risk degree as follows:

$$\min \Gamma = \frac{1}{t} \sum_{\tau=1}^{t} [RD(\tau) - RD]^2 \quad (22)$$

$$\begin{cases} 0 \leq \alpha_{p,k} \leq 1 \text{ and } \sum_{p=1}^{n} \alpha_{p,k} \leq 1 \\ w_i^- < w_i < w_i^+ \\ h_{i,k}^- < h_{i,k} < h_{i,k}^+ \end{cases} \quad (23)$$

where w_i^+ and w_i^- denote the upper and lower bounds of evidence weight w_i, respectively. $h_{i,k}^+$ and $h_{i,k}^-$ represent the upper and lower bounds of referential value $h_{i,k}$ in initial FoDs, respectively. The goal is to minimize Γ, and the expected risk degree RD is given by experts.

2.6. Summary of the Risk Assessment Algorithm

Based on the above analysis, the flow chart of the risk assessment algorithm is shown in Fig. 2.

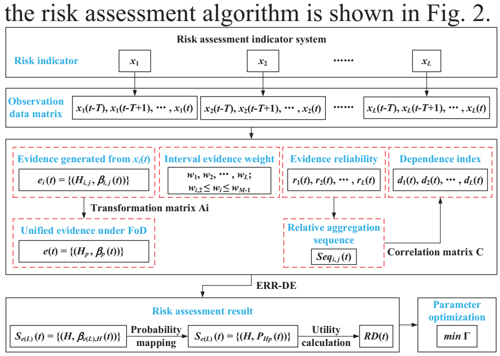

Fig. 2. Flow chart of risk assessment algorithm.

Step 1: Construct the risk assessment indicator system, as shown in Fig. 1.

Step 2: Obtain the observation data matrix of each sub indicator, denoted by $\mathbf{Z}_i(t) = [x_i(t-T) \quad x_i(t-T+1) \quad \cdots \quad x_i(t)]$.

Step 3: Generate the initial evidence from the observation data $x_i(t)$ as $e_i(t)$ in (4), and transform $e_i(t)$ to $e(t)$ as (5) using (6) and (7).

Step 4: Obtain the interval weight of each piece of evidence from experts as $w_i \in [w_{i,2}, w_{i,M-1}]$ in Section 2.2.

Step 5: Calculate the reliability of each piece of evidence as $r_i(t)$ based on static reliability and dynamic reliability using (8).

Step 6: Determine the relative aggregation sequence $Seq_{i,j}(t)$ of evidence based on (9), and calculate the dependence index of evidence based on (10)-(14).

Step 7: Recursively aggregate all the evidence using the ERR-DE model in Section 2.4, and the risk assessment results $S_{e(L)}(t)$ can be obtained as (19).

Step 8: Map $S_{e(L)}(t)$ to real probabilities as (20), and calculate the risk degree of complex systems as (21).

Step 9: Establish the parameter optimization model as (22) and (23), and use typical optimization algorithms to generate the optimal parameters.

3. Case Study

In this section, the risk assessment of a type of laser gyroscope is conducted to show the implementation process of the proposed ERR-DE model. The risk of laser gyroscope is mainly related to drift coefficient and light intensity, and the indicator system can be constructed as Fig. 3.

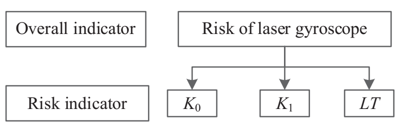

Fig. 3. Risk assessment indicator system of laser gyroscope.

In Fig. 3, K_0 and K_1 denote the zero-item and first-item drift coefficients of the laser gyroscope, respectively. LT is the light intensity. In the experiment, the gravitational acceleration of the experimental site is 9.8015 m/s^2. A total of 100 sets of data are collected, as shown in Fig. 4.

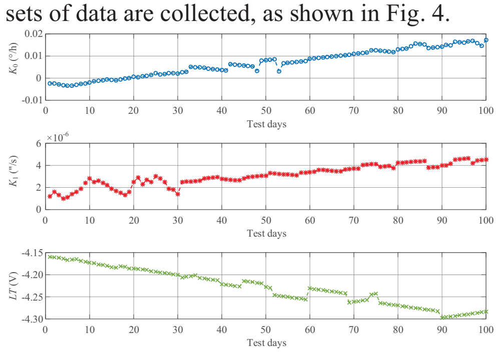

Fig. 4. Observation data of K_0, K_1 and LT.

3.1. *Acquisition of Evidence and Determination of Initial Parameters*

Based on the expert knowledge, the referential grades and referential values of risk indicators are listed in Table 1 and Table 2.

Table 1. Referential grades and referential values of drift coefficients.

Referential grade	K_0 (°/h)	K_1 ("/s)
Small	[-0.03, -0.01]	[0, 3×10^{-6}]
Medium	[-0.01, 0.01]	[3×10^{-6}, 7×10^{-6}]
Large	[0.01, 0.03]	[7×10^{-6}, 1.2×10^{-5}]

Table 2. Referential grades and referential values of light intensity.

Referential grade	LT (V)
Very weak	[-4.32, -4.28]
Weak	[-4.28, -4.24]
Medium	[-4.24, -4.2]
Strong	[-4.2, -4.16]
Very Strong	[-4.16, -4.12]

According to Table 1, the FoD of drift coefficients is $\Theta_1 = \{\text{Small}, \text{Medium}, \text{Large}\}$. The unit of light intensity is V in Table 2, which is the output of an A/D conversion circuit. As such, the FoD of light intensity is $\Theta_2 = \{\text{Very weak}, \text{Weak}, \text{Medium}, \text{Strong}, \text{Very strong}\}$. By using the utility-based equivalence transformation technique, all the observation data can be transformed to evidence. Limited by page number, the detailed results will not be presented here.

To obtain the evidence weight, we invite 5 experts to participate in empowerment. Also, the FoD of risk assessment result is set as $\Theta = \{\text{Low}, \text{Medium}, \text{High}\}$. Obviously, there is inconsistency among Θ_1, Θ_2 and Θ. Thus, the evidence in Θ_1 and Θ_2 should be transformed to Θ. The initial evidential parameters are given as follows for illustration:

Table 3. Initial parameters of risk indicators.

Risk indicator	Interval weight	Static reliability	Discounting factor
K_0	[0.8, 0.9]	0.95	0.8
K_1	[0.75, 0.85]	0.85	0.8
LT	[0.75, 0.85]	0.9	0.8

Table 4. Referential values of risk grades in Θ.

Risk grade	Low	Medium	High
Referential value	[0, 0.4]	[0.4, 0.7]	[0.7, 1]

$$\mathbf{A}_1 = \begin{bmatrix} 0.25 & 0.5 & 0.25 \\ 1 & 0 & 0 \\ 0.8 & 0.2 & 0 \end{bmatrix} \tag{24a}$$

$$\mathbf{A}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0.8 & 0.2 & 0 \\ 0.25 & 0.25 & 0.5 \end{bmatrix} \tag{24b}$$

$$\mathbf{A}_3 = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 1 & 0 & 0 \\ 0.8 & 0.2 & 0 \\ 0.6 & 0.4 & 0 \\ 0.4 & 0.3 & 0.3 \end{bmatrix} \tag{24c}$$

As such, all evidence can be unified to the same FoD as Θ. By using the distanced-based method, the dynamic reliability of each risk indicator can be calculated. Based on (8), the evidence reliability is shown in Fig. 5.

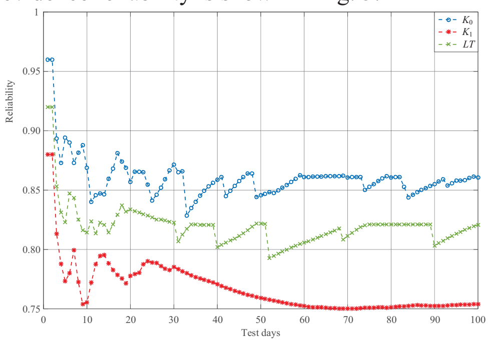

Fig. 5. Reliability of risk indicators.

Based on (9)-(14) and Fig. 4 and Fig. 5, the significance level is set as 5%, and the RTDC value of each risk indicator can be obtained. It is calculated that $d_1 = 1$, $d_2 = 0.3496$, $d_3 = 0.5106$.

3.2.Parameter Optimization

Based on (22), the objective function can be established, and the constraints are obtained from Table 1 to Table 4.

$$min\ \Gamma = \frac{1}{100}\sum_{\tau=1}^{100}[RD(\tau) - RD]^2 \tag{25}$$

In this section, the initial values of all evidential parameters are set as the average values of corresponding intervals. Using the Fmincon Function in Matlab Optimization Tool, the above optimization problem can be solved. The optimal transformation matrices are as follows:

$$\mathbf{A}_1^* = \begin{bmatrix} 0.56 & 0.26 & 0.18 \\ 0.87 & 0.08 & 0.05 \\ 0.05 & 0.13 & 0.82 \end{bmatrix} \tag{26a}$$

$$\mathbf{A}_2^* = \begin{bmatrix} 0.56 & 0.25 & 0.19 \\ 0.25 & 0.28 & 0.47 \\ 0.34 & 0.32 & 0.34 \end{bmatrix} \tag{26b}$$

$$\mathbf{A}_3^* = \begin{bmatrix} 0.22 & 0.29 & 0.49 \\ 0.19 & 0.24 & 0.57 \\ 0.30 & 0.41 & 0.29 \\ 0.73 & 0.17 & 0.10 \\ 0.40 & 0.31 & 0.29 \end{bmatrix} \tag{26c}$$

Other optimal evidential parameters are listed in Table 5 to Table 8.

Table 5. Optimal referential values of drift coefficients.

Referential grade	Small	Medium	Large
K_0 (°/h)	-0.0230	0.0009	0.0132
K_1 (''/s)	1.9×10^{-6}	4.2×10^{-6}	9.5×10^{-6}

Table 6. Optimal referential values of light intensity.

Referential grade	LT(V)	Referential grade	LT(V)
Very weak	-4.302	Strong	-4.185
Weak	-4.265	Very Strong	-4.137
Medium	-4.216		

Table 7. Optimal weight of risk indicators.

Risk indicator	K_0	K_1	LT
Optimal weight	0.85	0.80	0.80

Table 8. Optimal referential values of risk grades in Θ.

Risk grade	Low	Medium	High
Optimal Referential value	0.0157	0.5564	0.9826

3.3.Risk Assessment

Based on Section 2.4, the risk probabilities of laser gyroscope relative to different risk grades can be obtained using the optimized ERR-DE model, shown as Fig. 6.

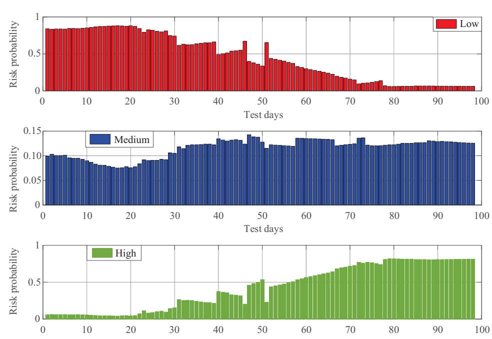

Fig. 6. Risk probability of laser gyroscope.

It can be seen from Fig. 6 that as the number of test days increases, the risk probability of grade “Low” decreases dramatically from roughly 0.88 to 0.06. For grade “Medium”, the risk probability fluctuates around 0.10, and the amplitude is relatively gentle. In comparison, the risk probability of grade “High” increases rapidly from about 0.04 to 0.82. This also means that the likelihood of risks occurring with laser gyroscope is increasing. It should be noted that the trend of risk probability change for three risk grades is not strictly monotonically increasing or decreasing.

To further reveal the overall changes in the risk degree of laser gyroscope, the risk assessment results between the optimized ERR-DE model and the expected result are compared, as shown in Fig. 7.

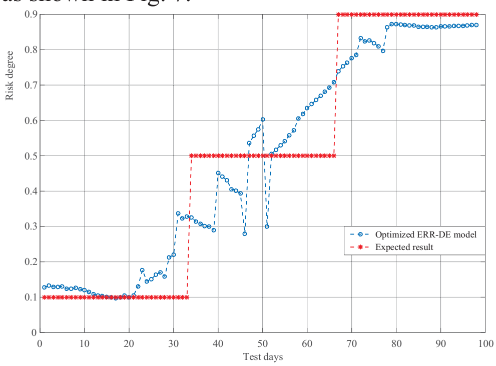

Fig. 7. Comparison between the ERR-DE model and the expected result.

According to Fig. 7, the mean square error (MSE) value between the optimal assessment result by ERR-DE model and the expected result is 0.0079. This demonstrates that the optimized ERR-DE model can well describe the changes in the risk degree of laser gyroscope. Overall, the risk degree increases from about 0.10 to 0.87. As mentioned in Section 2.4, an increase in $RD(t)$ means an increase in risk degree, which is also consistent with Fig. 6. Thus, the effectiveness of the proposed ERR-DE model is validated.

3.4.Comparison with Classical ERR

In this section, classical ERR model is used for comparison to further verify the effectiveness of the proposed ERR-DE model. In ERR, the inconsistency of FoD is not considered, and the dependence index of evidence is ignored. To unify the FoD of evidence, we set $\Theta_2 = \{\text{Weak}, \text{Medium}, \text{Strong}\}$, and the referential values of LT are listed in Table 9. Other parameters are consistent with ERR-DE.

Table 9. Referential grades and referential values of light intensity in ERR scheme.

Referential grade	Weak	Medium	Strong
LT (V)	[-4.3, -4.25]	[-4.25, -4.2]	[-4.2, -4.15]

Using the Fmincon Function in Matlab Optimization Tool, the ERR model can be optimized. The risk assessment results are compared in Fig. 8.

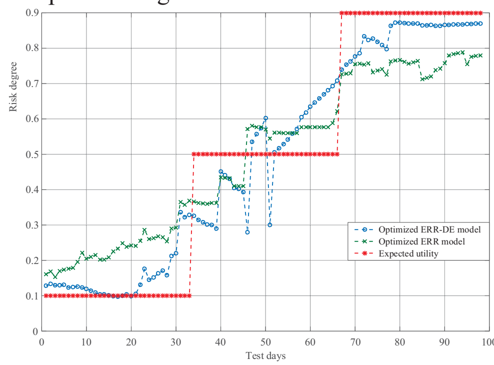

Fig. 8. Comparison among different ER models.

It can be seen from Fig. 8 that the MSE value between the optimal assessment result by ERR model and the expected result is 0.0172. Compared with ERR model, the risk assessment accuracy of ERR-DE model increases by about 54.7%. Due to the incomplete consideration of uncertainty in ERR model, its risk assessment accuracy is relatively low. On the one hand, the inconsistency of FoD is not considered in the ERR model, increasing the uncertainty in setting initial values. On the other hand, in the ERR model, all the evidence is independent of each other in default, which is difficult to match with reality. Hence, the above comparative study shows the effectiveness of the proposed ERR-DE model.

4. Conclusion

In this paper, an ERR-DE-based risk assessment model for complex systems is briefly introduced with a case study. It constitutes a multi-source information fusion framework, which can make full use of qualitative knowledge and quantitative data to analyze the system risk under uncertainties. Based on the establishment of a general risk assessment indicator system, all risk indicators are transformed to evidence with different FoDs. The transformation matrix is introduced to unify different evidence to the same FoD as assessment results. The subjective and objective evidential parameters are considered, and the dependence index of evidence is measured by RTDC. All the evidence is aggregated by the ERR-DE model to assess the system risk, measured by the risk probability and the risk degree. Moreover, the parameter uncertainty is well alleviated by a parameter optimization model.

In view of the achieved new results, future work should be focused on the sensitivity analysis or robustness analysis of the risk assessment model. Besides, the ability of ERR-DE model to deal with multi-scale evidence information needs to be further investigated.

Acknowledgement

The paper presents the results of the work granted by the National Natural Science Foundation of China under Grants 62303474.

Disclosure of Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

Fu, C., Hou, B, B., Xue, M., et al (2023). Extended belief rule-based system with accurate rule weights and efficient rule activation for diagnosis of thyroid nodules. IEEE Transactions on Systems, Man, and Cybernetics: Systems, **53**(1), 251-263.

Gizem, E., Sukru, I, S., and Emre, A., et al (2023). Operational risk assessment of ballasting and de-ballasting on-board tanker ship under FMECA extended Evidential Reasoning (ER) and Rule-based Bayesian Network (RBN) approach. Reliability Engineering and System Safety **231**, 108975.

He, R, F., Zhang, L, M., and Tiong, R, L, K (2023). Flood risk assessment and mitigation for metro stations: An evidential-reasoning-based optimality approach considering uncertainty of subjective parameters. Reliability Engineering and System Safety **238**, 109453.

Siami, M., Naderpour, M., and Ramezani, F., et al (2024). Risk assessment through big data: an autonomous fuzzy decision support system. IEEE Transactions on Intelligent Transportation Systems, **25**(8), 9016-9027.

Székely, G, J., Rizzo, M, L., and Bakirov, N, K (2007). Measuring and testing dependence by correlation of distances. The Annals of Statistics **35**(6), 2769-2794.

Tang, S, W., Zhou, Z, J., and Hu, C, H., et al (2022). A new evidential reasoning rule-based safety assessment method with sensor reliability for complex systems. IEEE Transactions on Cybernetics **52**(5), 4027-4038.

Yager, R, R (2009). On the fusion of non-independent belief structures. International Journal of General Systems **38**(5), 505-531.

Yang, J, B (2001). Rule and utility based evidential reasoning approach for multiattribute decision analysis under uncertainties. European Journal of Operational Research **131**(1), 31-61.

Yang, J, B., and Xu, D, L (2013). Evidential reasoning rule for evidence combination. Artificial Intelligence **205**, 1-29.

Yang, J, B., and Xu, D, L (2025). Maximum likelihood evidential reasoning. Artificial Intelligence **340**, 104289.

Zhang, P., Zhou, Z, J., and Tang, S, W., et al (2023). On the evidential reasoning rule for dependent evidence combination. Chinese Journal of Aeronautics **36**(5), 306-327.

Zhao, F, J., Zhou, Z, J., and Hu, C, H., et al (2018). A new evidential reasoning-based method for online safety assessment of complex systems. IEEE Transactions on Systems, Man, and Cybernetics: Systems **48**(6): 954-966.

Zhou, Y., Zhang, Z, Y., Fan, K, F., et al (2025). A novel quantitative risk assessment model for industrial control systems integrating the cyber-physical domain. IEEE Transactions on Industrial Informatics, **21**(8), 6433-6442.