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Resilience Performance Modelling using PT Delay Elements

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Resilience is mostly represented using a curve of performance. There are multiple mathematical resilience performance representations. When subjected to external forces or disturbances, infrastructure systems often exhibit a delayed response, the characteristic of what is already mentioned in the cited works. If resilience is understood as a dampening factor to some disturbance, it is suitable to model it phenomenologically using delay elements. A delay element basically is the functional description (transfer function) between an input and output relation. The input describes the presence of a disturbance, which is mostly a step function, and following that the delay element could be used to describe the delayed system response. It is important to mention that the decay and recovery in performance are generally not able to be modelled using the same transfer function, because in most real cases the decay happens much faster than the repair process. Therefore, the behaviour needs to be either directional, or time dependent. We apply the concept of modelling a resilience process using delay elements to the use-case of a power outage in a factory. The availability of electricity serves as the input step function for this scenario. The output of the model describes the system performance, such as the number of parts produced or sold by the factory. After a sudden power outage, depending on the circumstances, the factory may be able to continue producing or selling for a shorter or longer period. This represents the system's response to the power failure. Once the disruption is resolved, production and sales can resume, albeit with a delay.

Keywords: resilience management, infrastructure, modelling.

1. Introduction

Modern technical systems are increasingly complex, interconnected, and exposed to a wide range of disturbances, including component failures, environmental impacts, and intentional attacks (Crowther & Haimes, 2010). As a result, ensuring not only reliable operation but also the ability to respond to and recover from disruptions has become a critical design and assessment objective. Resilience engineering considers the dynamic behaviour of a system before, during, and after a disruptive event (Patriarca et al., 2018) and thus extends classical concepts such as reliability and robustness, which primarily focus on failure avoidance (Woods, 2018). A systematic understanding of (socio-) technical resilience is therefore essential for the development, evaluation, and operation of systems that must maintain functionality and performance under uncertain and evolving conditions.

2. Background

A resilient system is one that maintains state awareness and an accepted level of system operationality in response to disturbances, including all threats of unexpected and malicious nature (Rieger et al., 2009). To improve the resilience of a system, an important step is to develop methods and techniques to properly quantify relevant resilience metrics (Linkov & Trump, 2019). This supports decision makers to evaluate and compare the different options of resilience enhancement and to determine the best measures. In the following, we propose and use a new perspective on the resilience curve by assessing the resilience of a system consisting of delay elements.

2.1 The resilience curve

The resilience curve is the most famous model for engineers to explain, measure and quantify resilience (Mentges et al., 2023). While it is well recognized that resilience is more than just recovery from stress or functioning under pressure, the curve remains highly used (Eisenberg et al., 2025).

The curve was originally introduced by Bruneau et al. (2003) in combination with the quantification of the resilience loss as

$$RL = \int_{t0}^{t1} [100 - Q(t)]\mathrm{d}t \tag{1}$$

$Q(t)$ describes the system performance (or quality). The integral boundaries t_0 and t_1, representing the beginning and end of the resilience cycle, must be defined by the assessor.

Proposals for fitting mathematical models to the curve are for example given by Cimellaro et al., 2010; Demmer et al., 2022; Stolz et al., 2024. A detailed literature review regarding other mathematical models is given in Yodo and Wang (2016), Poulin and Kane (2021) and Tang et al. (2023).

2.2. Delay Functions

We use PT_n-elements (n-th order delay elements) to model a system during a disturbance. The concept of PT reactions is rooted in the fundamental principles of control theory and system dynamics. These linear time-invariant (LTI) systems consist of a gain factor that scales the input signal (P), and a series of n first-order time delay elements (T), each introducing a certain inertia or lag in the system response. When describing a system as LTI, this means that the output depends only on its current input and internal state variables without any memory of past inputs or outputs. For example, a first-order lag system like PT_1 responds to a step change in input with an exponential rise time, reaches 63.2% (or $1/e$) of its final value within one time constant (τ) and decays exponentially back to zero after the step is reverted.

The transfer function of a PT_1 system (first order system) is represented mathematically with Equation (2) and PT_2 with Equation (3), where $G(s)$ is the transfer function, K is the gain or proportionality constant, T represents the time delay or time constant, s is the complex frequency and D is the damping factor.

$$G(s) = \frac{K}{Ts + 1} \tag{2}$$

$$G(s) = \frac{K}{T^2s^2 + 2DTs + 1} \tag{3}$$

As already mentioned, the PT_1 system's response to a step input is characterized by an exponential rise or decay. The PT_n response, that is a series connection of multiple PT_1 elements, approaches a logistic behavior. The delay is influenced by the gain factor K and the damping factor D.

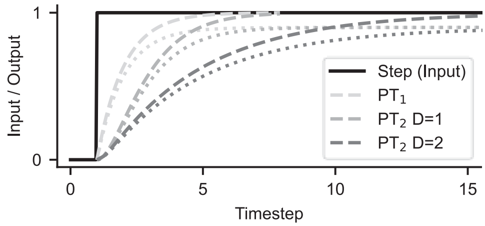

Figure 1. Input step function and system reaction with various PT reactions for gain factor K=1. The dotted lines represent the K=0.9 reactions.

PT elements are often used in control theory to model any kind of delayed processes. The step function describes the presence of a disturbance and a PT element describes the delayed system response. Especially, if resilience is understood as a dampening factor to some disturbance, it is suitable to model it using a PT element. Indeed, the resulting performance curve from a PT-model resembles the idealized shape of a resilience curve. For real systems, the shape can be really diverse and is most likely defined by the system. However, for our example PT-model reactions give enough flexibility to model a resilience assessment scenario. The effect of delays within (control) systems is also discussed in Demmer et al. (2025).

Additionally, the approach to model resilience via PT-elements respects the linkage between the intensity of a disturbance and the intensity of its effects. The maximum performance loss, sometimes called vulnerability (Häring et al., 2016), is defined by the gain K. Hence, the loss of resilience (Equation (1)) depends on the intensity of the disturbance.

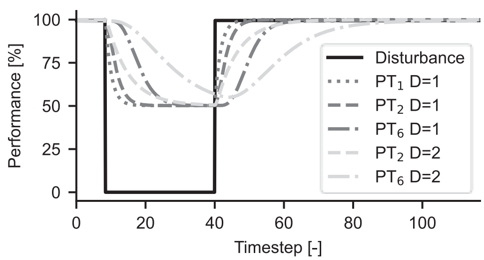

Figure 2. Arbitrary resilience process modelled using various PT reactions to a sudden disturbance. The gain parameter K is 0.5 for the PT_2 reactions and 0.8 for the PT_6 reactions depicted here.

In Figure 2 the decay and recovery in performance are modelled with the same function. However, in most real cases, the decay happens much faster than the repair process. While the breakdown occurs quickly and may be best modelled with using a PT_1 reaction, repairing such complex equipment takes considerably longer. This may be better expressed with a higher order like PT_6 or a higher damping factor D. Therefore, the system reaction should be modelled with respect to the direction.

Another use of PT behavior is the activation of a backup system, as redundancy can help increasing resilience (Jackson & Ferris, 2013). When the main system collapses, the overall system performance depends on how fast the backup can be activated (see Figure 3 for an example). In that case the down-time is only depended on the time the backup is activated and the time it takes to activate the system.

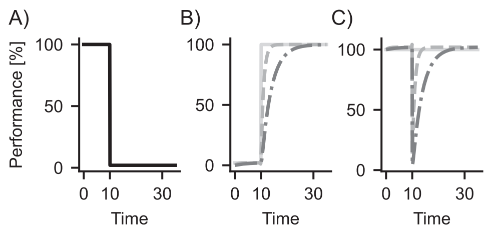

Figure 3. A): The main system breaks down. B): Three different backup systems are implemented. The solid line marks an instantly available backup source, while the grey lines represent a PT-1 behaviour (T=1) and a PT-2 behaviour (T=1, D=2). C): The resulting overall system performance (A+B=C) depends on the type of the backup source in B).

2.3. Uncertainties

An important aspect of analyzing a system's resilience is accounting for variability in the scenarios under consideration. When a system's performance drops due to unexpected disruptions, it is essential to also consider uncertainties regarding the subsequent evolution of performance. Only a few works consider that the resilience performance curve is subject to many uncertainties. Decò et al. (2013) state that every point of the performance curve during a disruption is associated with uncertainties regarding time of occurrence or performance level. Regardless of the method by which the data of the performance curve is gathered (via simulations or field-tests), there are always aleatoric and/or epistemic uncertainties that ought to be considered.

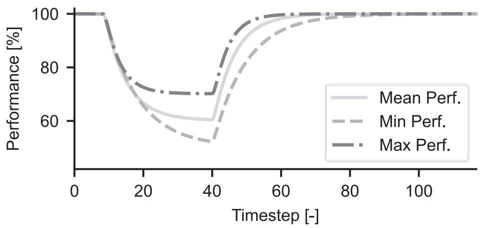

Figure 4. Every predictive performance chart should include the involved uncertainties, for example by showing the mean performance besides the minimum and maximum expected performance. Using a PT_n model, this is achieved using various factors K and D.

Epistemic uncertainty may decrease with additional data, whereas aleatoric uncertainty is irreducible (Kiureghian & Ditlevsen, 2009).

3. Application

We apply this concept to the use-case of a partial power outage in a factory. After a sudden shutdown of part of the machinery, the factory may be able to continue producing and selling products partially. The reduced production capacity represents the system's response to the power failure. Once the disruption is resolved, production and sales can resume, albeit both happening with a delay.

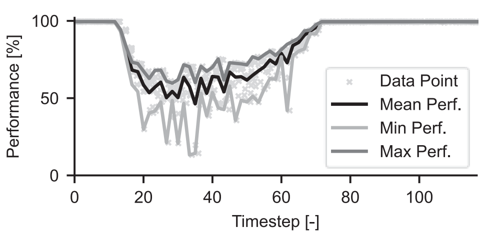

Figure 5. Performance data after different power outages.

We consider the availability of electricity as the input step function for this scenario and the system performance (the number of parts produced or sold by the factory) as output of the model.

When considering the system to have a PT_n behavior, the first step is to identify the order of the lag in the system. The input function is a step-function from 100 to 0 at $t = 10$ which returns to 100 at $t = 40$ (Figure 6). We consider at first the mean performance after a power outage. The system performance over the whole resilience cycle can be approximated by a second-order transfer function (PT_2) with a damping ratio of $D = 4$ (see Figure 6).

However, it is significantly more accurate to consider the first and second half of the resilience cycle separately. Due to its delay until the drop begins, the decay is better modelled with PT_6 and $D = 1$, while the following recovery's best fit is a PT_6 with $D = 2$. The magnitude of the response is described with $K = 0.5$ for the PT_2 reaction and $K = \sqrt[3]{0.5} = 0.8$ for the PT_6 reaction.

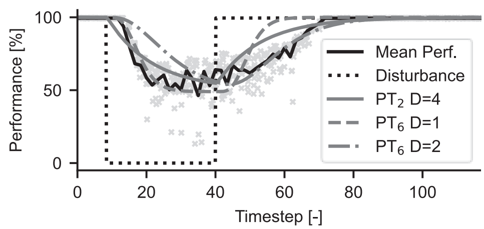

Figure 6. Fitting various delay functions to the given data.

Following that, in order to get a more precise curve fit, the raw data has to be split up into breakdown (Phase 1, 2) and recovery (Phase 3, 4) (see Figure 7 for the result). This allows to use the $D = 1$ reaction for the first half of the cycle and the $D = 2$ reaction for the second half and results in the better fit.

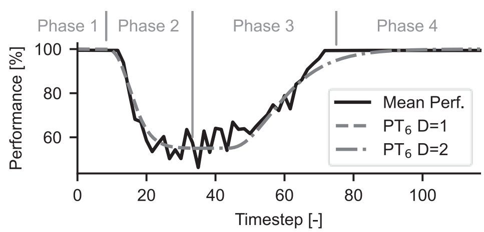

Figure 7. Delay fitting for mean performance. Phase 1 and 2 is best described using a PT_6 with D_1=1, while phase 3 and 4 is best described using PT_6 D_2=2.

Subsequently, the identified system behaviour can be adjusted according to the given uncertainties (Figure 5) by adjusting parameters K and D. For the given case, $D_1 = 1$ and $D_2 = 2$ is suitable for describing the mean as well as the maximum and minimum performance scenario. The difference is only in the gain factor, where $K = 0.78$ describes the mean performance (Figure 7), $K = 0.87$ the minimum performance (worst-case) and $K = 0.7$ the maximum performance (best-case) (Figure 8).

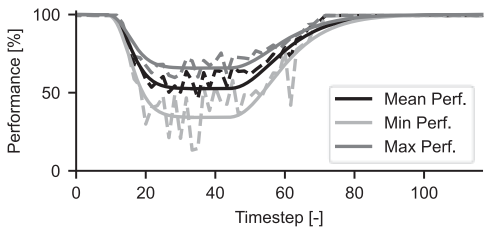

Figure 8. Bands of uncertainty for the system. All three curves are PT6 with D_1=1 and D_2=2, as in Figure 7.

The three performance charts (Figure 8) describe the system behaviour more accurately than the average curve. They form the basis for the following resilience assessment according to Equation (1).

The resulting loss of performance (or resilience loss RL) varies significantly between the three considered scenarios (Figure 9). The absolute difference between the minimum and the maximum resilience loss is 2259, but more importantly the relative difference is at 92% which means that the outcome in terms of resilience loss is not really predictable.

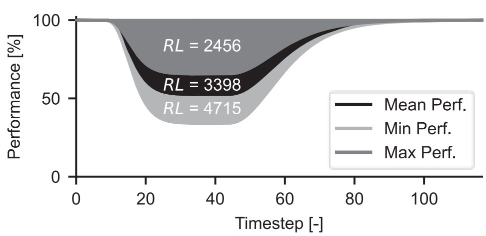

Figure 9. Calculated resilience loss (RL) according to Equation (1) for t_0=0 and t_1=100

4. Discussion

The delayed system's reaction to a sudden disturbance results in a performance curve similar to the ones used in resilience engineering (see e.g. Ayyub, 2014, Fig. 3; Mentges et al., 2023, Fig. 1; Poulin & Kane, 2021).

Including uncertainties is crucial to every safety, reliability or resilience assessment. The absolute and relative differences are crucial additional information to avoid giving a false sense of certainty regarding the results.

Our results show that the absolute loss of resilience RL is subject to high uncertainty and therefore not predictable. However, this does not mean that countermeasures are not useful. This aspect is not considered here, but safer and more resilient subcomponents may improve the overall system performance in all scenarios, although quantifying the exact effect is difficult.

Additionally, we considered only the overall performance of a system composed of multiple subsystems. This curve may be derived from the individual subsystem curves and provides only a first impression and general overview of the system's behaviour. To fully understand the details, it is essential to examine the curves of the relevant subsystems to identify the root cause of the performance decline, learn from this issue, and implement effective countermeasures to prevent it from happening again.

Modelling the infrastructure response to disturbances in terms of performance with PT elements is a strong simplification of complex system dynamics and more complex scenarios might require the use of higher-order models or different mathematical frameworks to accurately capture their dynamics. However, this approach allows to analyse how the system responds to various inputs and disturbances. While the model draws on previous events, it is important to recognize that future behaviour may differ from the past.

Splitting the data points into resilience phases 1 & 2 and 3 & 4 was a manual process here. It is conceivable that this process could be automated in the future. However, it will always be necessary to define how many individual processes are involved e.g. whether sub-steps are included, because maybe the disturbance is not a clear step function as assumed here (see Figure 6). One may as well use these models for design optimization, stability analysis, or predictive maintenance applications. This is not included here but an interesting extension for future work.

5. Conclusion

We introduced a novel approach to understanding the resilience curve as a series of coupled delay elements (PT_n) demonstrating that the curve reflects delays within a system that are crucial for understanding its meaning.

Even though the quantified loss of resilience was highly uncertain, this does not mean diminish the value of the results. Indeed, the method could be used to understand the systems better which helps to implement the measure with the biggest advantage.

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