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# Interaction Effects in Subinterval Sensitivity Analysis

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This paper expands upon subinterval sensitivity by considering the simultaneous effect of pairs and triplets of variables at the same time.
We assert that individual subinterval sensitivity indices quantify interaction effects and thus they can be regarded as full-blown global sensitivity indices. We consider simultaneous effects only as a means to show the global character of individual indices. In fact, the calculation of simultaneous effects is used as a diagnostic tool to detect the presence and extent of interaction effects. This confirms our proposition that individual subinterval indices are indeed comparable to Sobol' total indices.
It turns out that when there are interaction effects the individual indices lose "additivity", meaning that their sum is never equal to their corresponding simultaneous indices. In particular, we find that simultaneous indices are always smaller than the sum of the individual first-order indices in the presence of interaction effects. Moreover, we find that the exact opposite is true also: When the first-order indices add up exactly to their corresponding simultaneous index there are no interaction effects.
This method can be very cheap in determining interaction effects because only a small number of subintervals (e.g. five) are needed.
In the absence of interaction effects, the difference between the sum of first-order indices and their corresponding simultaneous index can also quantify the effect of inflation due to repeated variables within a function.
The caveat is that the conflated presence of inflation and interaction effects within a mathematical model contaminates the results, making it difficult to discern between the two effects and in turns adding artefactual uncertainty on the sensitivity indices. This motivates the need to develop ways of removing or mitigating inflation of interval computation to ensure that reliable results are obtained.

*Keywords*: Sensitivity analysis, Interaction effects, Interval computation, Subinterval sensitivity.

## 1. Introduction

Sensitivity analysis can be grouped into two sets of methods: local sensitivity analysis where output variability is considered around specific nominal values; and global sensitivity analysis, which considers variations across the entire input domain (Iooss and Saltelli, 2017). Global sensitivity analysis, through analysis of all factors at the same time, is a preferable approach for nonlinear models due to the possible presence of interaction effects. Interaction effects can be defined as the effect of one control variable being dependent on the value of another control variable (Mize, 2019). Interaction effects can be a significant source of variable importance and can result in type II errors (non-identification of important variables) if ignored (Saltelli et al., 2008).

This paper aims to expand upon subinterval sensitivity analysis, an interval-based non-probabilistic method introduced in (Miralles-Dolz et al., 2022) and further developed in (Ochnio and de Angelis, 2025), by introducing *simultaneous* indices, i.e. higher-order sensitivity indices obtained subintervalising two or more variables of interest at the same time. By analysing multiple variables of interest at the same time, we can compare the individual effects against the corresponding simultaneous effects and draw conclusions about their interactions.

## 2. Background

### 2.1. *Interaction effects within sensitivity analysis*

The quantification of interaction effects is one of the key attributes of global sensitivity analysis methods that distinguishes them from local methods. By knowing which variables do or do not have interaction effects, we also gain a better understanding of which variables can be changed freely without worrying about their dependencies with other variables.

One of the clearest distinctions of interaction effects is present in Sobol' indices, introduced in (Sobol, 1993), with its distinction between the main and total sensitivity indices. The main effect index within Sobol' indices is the effect of only the analyzed variable on the model, without considering any other variables. The total effect index includes both the main effect index and the effects of any interactions between the analyzed variable and all other variables in the model.

Another method which can indicate the presence of interaction effects is the Elementary Effects (EE) method, introduced by (Morris, 1991) and further refined in (Campolongo et al., 2007). A key note is that this method does not quantify interaction effects, instead screening out which variables have non-linearity or interaction effects. This method uses two values: the mean for the importance of the variable; and the standard deviation to detect if a variable is either non-linear or has interactions with other variables.

To the best of authors' knowledge, there is no instance in the present literature which discusses or quantifies interaction effects within subinterval sensitivity analysis, which motivates this paper.

### 2.2. *Subinterval sensitivity*

Let $f : \mathbb{R}^d \to \mathbb{R}$ be the model, $x = x_1, ..., x_d$ be a bounded vector of inputs, $[\underline{x}, \overline{x}]$ some fixed bounds of interest such that $x \in [\underline{x}, \overline{x}]$, and $\mathsf{f}$ an interval extension of $f$ such that $[\underline{y}, \overline{y}] = \mathsf{f}([\underline{x}, \overline{x}])$, where $\mathsf{f}$ is evaluated using the rules of interval arithmetic. After partitioning the $i$-th input into $N$ tiling subintervals such that $[\underline{x}_i, \overline{x}_i] = \cup_n^N [\underline{x}_i, \overline{x}_i]_n$ and evaluating the interval extension on each of them $[\underline{y}, \overline{y}]_{i,n} = \mathsf{f}([\underline{x}_1, \overline{x}_1], ..., [\underline{x}_i, \overline{x}_i]_n, ..., [\underline{x}_d, \overline{x}_d])$, the subinterval sensitivity index for the $i$-th input is

$$S_i := 1 - \frac{\sum_n^N (\overline{x}_i - \underline{x}_i)_n \left(\overline{y} - \underline{y}\right)_{i,n}}{\left(\overline{y} - \underline{y}\right)(\overline{x}_i - \underline{x}_i)}. \quad (1)$$

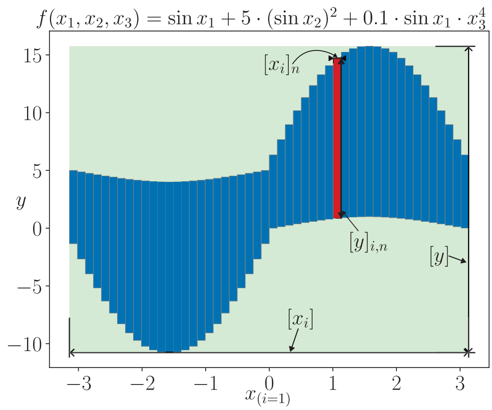


Fig. 1. Visual representation of Eq.(1) using $x_1$ from the Ishigami function where $N = 50$. The bounds of each subinterval are represented with blue rectangles, and the enclosing box in green. The sensitivity index for $x_1$ is 0.560. Subinterval area highlighted in red corresponds to $x_1$ at $n = 33$ where $x_1 = [1.005, 1.131]$ and $y_{33} = [0.844, 14.719]$.

The numerator in Eq. (1) is the sum of all subinterval areas (see overall area of sub boxes in Fig. 1) and the denominator is the area of the $x_i y$ graph enclosing box (see background box in Fig. 1). The sensitivity index $S_i$, ranges from 0 to 1 for all $i = 1, ..., d$. When $S_i = 0$, the partitioning has no effect, the numerator is equal to the denominator and so $y$ has no functional dependence on $x_i$. When $S_i = 1$, the subinterval areas are zero and so $y$ has full functional dependence on $x_i$.

It is worth noting that this method is unaffected by the curse of dimensionality as the computational cost scales linearly with the number of variables and subintervals being used.

## 3. Simultaneous indices

The concept of subinterval sensitivity can theoretically be extended to any simultaneous grouping of variables. For example, for pairs of variables the simultaneous index (order two) is

$$S_{ij} := 1 - \frac{\sum_m^M \sum_n^N \left(\overline{x}_i - \underline{x}_i\right)_n \left(\overline{x}_j - \underline{x}_j\right)_m \left(\overline{y}_{i,j} - \underline{y}_{i,j}\right)_{n,m}}{\left(\overline{y} - \underline{y}\right)\left(\overline{x}_i - \underline{x}_i\right)\left(\overline{x}_j - \underline{x}_j\right)}. \quad (2)$$

which has two variables of interest $x_i, x_j$ subintervalised simultaneously with $N$ and $M$ subintervals respectively. The sensitivity index $S_{ij}$ maintains the same range of 0 to 1 for all pairs of variables, which allows for direct comparison against individual indices.

Eq. (2) can be readily modified to include more simultaneous variables but with the caveat of adding more dimensions to the subinterval space partition. The addition of new dimensions does however increase the computational cost exponentially, making simultaneous indices computationally infeasible to calculate past fourth-order.

Fig. 2 shows the calculation of $S_{12}$ using Eq. (2) for the Ishigami function,

$$y = \sin(x_1) + a\sin^2(x_2) + bx_3^4\sin(x_1), \quad (3)$$

where $a = 5$ and $b = 0.1$ are used. Within the Ishigami function, the repetition of $x_1$ does not cause a dependency problem only where $b$ remains positive, otherwise the repetition of $x_1$ becomes problematic.

## 4. Interaction effects

Interaction effects occur when the effect of changing two factors is different from the sum of their individual effects (Saltelli, 2002). If we use the exact converse of this statement, namely that for non-interacting variables the effect of changing two factors is the same as the sum of their individual effects, we get the equation

$$S_i + S_j = S_{ij}. \quad (4)$$

This forms the basis for our main statement that is: subinterval sensitivity's interaction effects appear whenever Eq. (4) does not hold, i.e. whenever individual indices do not add to their simultaneous index. Of course for Eq. (4) to hold, full consistency is needed with the number of subintervals,

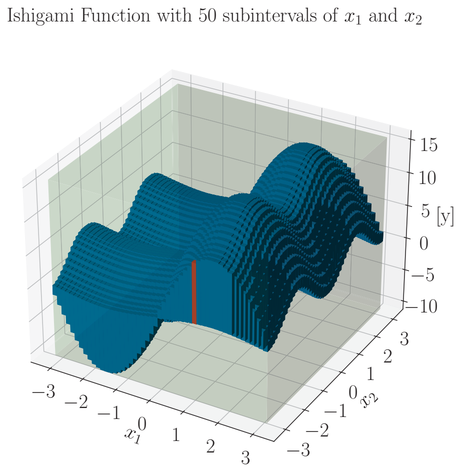


Fig. 2. Ishigami function, where $a = 5$ and $b = 0.1$, subinterval plot when $x_1$ and $x_2$ are subintervalised, with each subinterval volume in blue and the enclosing box in green. With $x_1$ and $x_2$ subintervalised, the sensitivity index is $S_{12} = S_{21} = 0.733$ for $N = M = 50$ subintervals. Subinterval volume highlighted in red corresponds to $x_1$ at $n = 33$ and $x_2$ at $m = 0$ where $x_1 = [1.005, 1.131]$, $x_2 = [-\pi, -3.016]$ and $y_{(33,0)} = [0.844, 9.797]$.

which must remain the same across all orders of subintervalisation.

### 4.1. *Linear function*

We first test our theory on a model with no interactions present to see that Eq. (4) holds. For this, we create a simple test function

$$y = x_1 + x_2 + x_3 + x_4 \quad (5)$$

that by visual inspection has no interactions present. Each variable in Eq. (5) is given the same range of $[2, 4]$ to ensure that no difference in sensitivity index can occur due to a different range of any variable. Each variable and second-order combination are subintervalised with $N = 50$ subintervals.

In Fig. 3 we can see that for each variable pair in Eq. (5), their individual indices are equal to the second-order simultaneous index, which matches our expectation from Eq. (4).

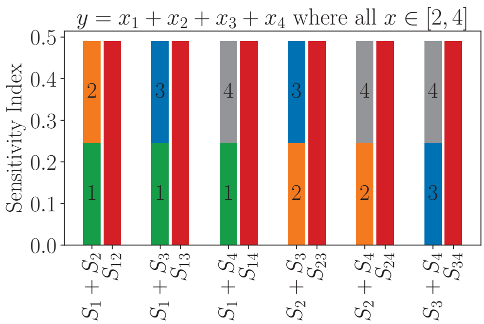


Fig. 3. Bar chart comparing sensitivity indices of each second-order combination of Eq. (5) against sum of the constituent variables indices with $N = 50$ subintervals for each variable. Sensitivity indices of $x_1$ in green, $x_2$ in orange, $x_3$ in blue, $x_4$ in grey, and higher-order combinations in red. All variables are in range $[2, 4]$.

### 4.2. *Non-linear function with interactions*

The claim is that Eq. (4) only holds for non-interacting pairs of variables, which requires us to test the theory against a function with known interaction effects. For the non-linear case, we have selected the Ishigami function which has a known interaction effect for the pair $(x_1, x_3)$.

Fig. 4 (left side plot) shows that Eq. (4) does not hold only for the pair $(x_1, x_3)$. This result not only corroborates our claim about indices' additivity in the presence of interaction effects, but also shows that interaction effects make the sum of individual indices greater than their corresponding simultaneous index, e.g. $S_1 + S_3 > S_{13}$ means that $x_1$, $x_3$ have interaction.

### 4.3. *Detecting interactions with low number of subintervals*

When we analyze multiple variables at the same time, the number of combinations that have to be analyzed is increased exponentially, which impacts the computational cost. However, if we are not interested in the accuracy of the results, we can reduce the number of subintervals used in subinterval sensitivity to reduce this cost while still detecting interaction effects.

Using the Ishigami function again, when we reduce the number of subintervals used from 50 to 5, we can reduce the computational cost by a factor of 10. In Fig. 4 (right side plot) we can see that despite the decrease in sensitivity indices for all variables, we can still successfully detect the interaction effects in the $(x_1, x_3)$ pair. As a result, we can very cheaply establish the presence of interaction effects between pairs or triplets of variables within a function by means of subinterval sensitivity.

## 5. Inflation

As far as interval arithmetic is concerned, the inflation of the output bounds caused by untracked dependencies is a major problem when propagating intervals through a function that has repeated variables. The additional unwanted width caused by inflation, often called artefactual uncertainty, can be problematic in subinterval sensitivity too. With Eq. (1) in mind, an increase to the outer bound width $\overline{y} - \underline{y}$ directly impacts the results we obtain by increasing the denominator and potentially decreasing the indices obtained. For this reason, knowing if there is inflation caused by repeated variables is very important as inflation can contaminate the results.

### 5.1. *Inflation detection tool*

The simplest way to detect inflation within any given variable is to subintervalise that variable and compare the bounds against the naive bounds before subintervalisation. We present this idea using a simple two variable function

$$y = 2x_1 + x_2 - x_1 \tag{6}$$

in which $x_1$ is repeated. The difference in subintervalising for $x_1$ and $x_2$ is clearly appreciated in Fig. 5. Subintervalising for the repeated variable $x_1$ results in a shrunk $[y]$ interval (left side plot), whereas for $x_2$ produces no effect (right side plot).

We can detect and quantify the inflation variable by variable by comparing the width of the subintervalised bounds against the naive bounds using

$$\mathrm{I}_i := 1 - \frac{\overline{y}_i - \underline{y}_i}{\overline{y} - \underline{y}}, \tag{7}$$

where an index of $0$ means the bounds are equal and there is no inflation. Thus, the inflation detec-

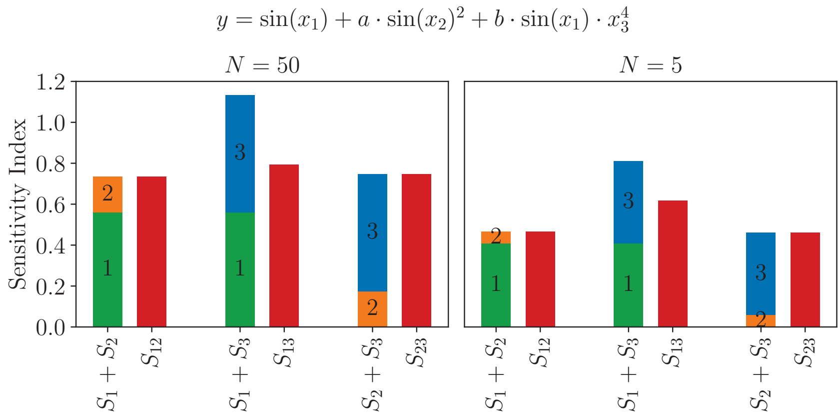


Fig. 4. Bar chart comparing the sensitivity indices of each variable pair in Eq. (3), where $a = 5$ and $b = 0.1$, against the sum of the constituent variables indices with $N = 50$ (left) and $N = 5$ (right) subintervals used for all variables being subintervalised. All variables in range $[-\pi, \pi]$.

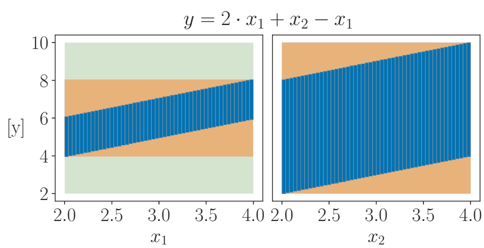


Fig. 5. Plots comparing the subintervalised output against the naive output bounds. Naive bounds are in green, subintervalised bounds in orange, and subintervals in blue. Both variables have the range $[2, 4]$ and $N = 50$ subintervals.

tion tool works as follows

$$\begin{cases} \mathrm{I}_i = 0 & \text{no inflation} \\ \mathrm{I}_i > 0 & \text{inflation.} \end{cases} \tag{8}$$

When inflation is present the index $0 < \mathrm{I}_i < 1$ quantifies the extent of inflation with respect to any specified variable. The use of this index is twofold: it can be used in a binary logic sense to screen out entirely the variables that cause no inflation at all, and it can be used in a quantitative sense to assess the extent of inflation and thus prioritise those variables that need further compute investment. In other words, first we split all variables into two groups, those that cause inflation and those that do not, and second we rank the variables belonging to the former group from higher to lower inflation.

For the two-variable function in Eq. (6), applying Eq. (7) with $N = 50$ subintervals gives inflation indices of $\mathrm{I}_1 = 0.49$ and $\mathrm{I}_2 = 0$. These results match what we would expect from visual inspection of Eq. (6), where the repeated $x_1$ produces an output width that is almost twice as large than expected, whereas $x_2$ causes no inflation at all.

### 5.2. *Inflation effect on subinterval indices*

Inflation can affect subinterval sensitivity indices. Unlike interaction effects, inflation can be removed in some functions by rearranging their expression in a single-use format. For example, Eq. (6) can be re-written as $y = x_1 + x_2$, which now has no repeated variables.

Table 1. Comparison of sensitivity and inflation indices for Eq. (6) and its single use version. Range for both variables is $[2, 4]$ with $N = 50$ subintervals used in both scenarios.

| Variable | Single use | | Repeated | |
|---|---|---|---|---|
| | $S$ | I | $S$ | I |
| $x_1$ | 0.49 | 0 | 0.480 | 0.49 |
| $x_2$ | 0.49 | 0 | 0.245 | 0 |

In Table 1 we can see how the sensitivity indices change for Eq. (6) depending on the expression used, either single-use or repeated expression. The index for $x_1$ goes down slightly despite the mitigated inflation from subintervalising the variable as a very small amount of inflation remains within each subinterval.

When we look at $x_2$, its sensitivity index is halved, which suggests that when a variable is inflated, it is actually the non-inflated variables whose importance is suppressed.

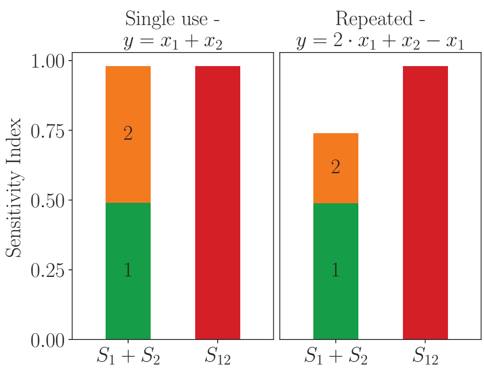


Fig. 6. Bar chart comparing the sensitivity indices of Eq. (6) in a single use expression and an expression with repeated variables causing inflation. Both variables have the range $[2, 4]$ and $N = 50$ subintervals.

In Fig. 6, we can see that in the single use expression, the sensitivity indices of Eq. (6) are additive. Whereas in the repeated-variables expression, the sum of the individual indices is actually less than their corresponding simultaneous index, further suggesting that the non-inflated variables have their indices suppressed in the presence of inflation.

## 6. Test case with interaction and inflation effects

By presenting an example single-use expression with interaction effects

$$y = (x_1 \cdot x_2 \cdot x_3) \cdot (x_4 + 1) \tag{9}$$

along with an equivalent version with variable repetition

$$y = (x_1 \cdot x_2 \cdot x_3 \cdot x_4) + (x_1 \cdot x_2 \cdot x_3), \tag{10}$$

we can see how introducing inflation effects into a function where interaction effects are already present affects the resulting sensitivity indices.

When Eq. (9) is used, all variables in the function have an inflation index of zero, which is expected as no variables are repeated. However, when Eq. (10) is used, $x_{1-3}$ each have an inflation index of $\mathrm{I} = 0.089$ while $x_4$ has $\mathrm{I} = 0$, showing that the re-written expression, despite being equivalent, introduces inflation effects when used.

The differences resulting from the change in how the expression is written can be clearly appreciated in Fig. 7 where the importance of $x_4$ is significantly reduced. Of added interest is the lack of notable reduction in importance for $x_{1-3}$, with each variable going from $0.245$ in Eq. (9) to $0.2445$ in Eq. (10). This happens despite all three of these variable having inflation, meaning that each variable should have been affected by the other variables' inflation.

## 7. Discussion

When we compare the sum of the individual sensitivity indices of the constituent variables against their simultaneous index, we can detect both interaction and inflation effects within subinterval sensitivity analysis.

For variables with no interaction or inflation effects, the sum of the individual indices is equal to the simultaneous index, which shows that the effects of each variable are directly translated when combining the effects with another variable. By contrast, when either interaction or inflation effects are present, the individual indices are no longer additive with respect to the simultaneous index. This shows that Eq. (4) is applicable to subinterval sensitivity and allows us to detect the presence of interaction or inflation effects.

We can use the non-probabilistic nature of the method to also decrease the number of subintervals being used to detect interaction or inflation effects while maintaining the rigor of the method. Even though we reduce the number of subintervals from $N = 50$ to $N = 5$, we are still able to correctly detect in the Ishigami function that $x_1$, $x_3$ interact while the other two pairs do not. This allows us to use subinterval sensitivity to detect

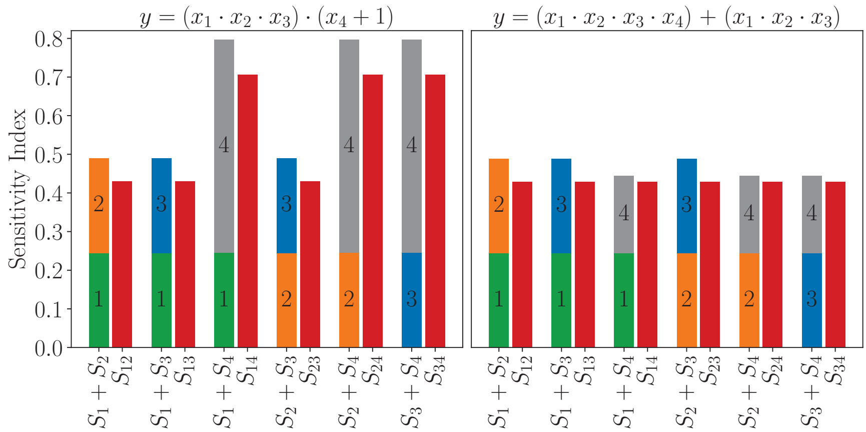


Fig. 7. Bar charts comparing the sensitivity indices using Eq. (9) (left) and Eq. (10) (right) while using the same variable ranges and number of subintervals in both scenarios. All variables have the range $[-2, -1]$ and $N = 50$ subintervals. Sensitivity indices of $x_1$ in green, $x_2$ in orange, $x_3$ in blue, $x_4$ in grey, and simultaneous indices in red.

variable interactions at a very low computational cost. Decreasing the computational cost cannot be matched by probabilistic methods, as the increased variability from the reduced coverage of samples would make it uncertain if an effect is truly present or if it is a sample specific result.

By comparison of Fig. 4 to Fig. 6, it is clear that interaction and inflation effects have opposite effects on the simultaneous index when compared to the sum of the individual sensitivity indices. In Table 2, we can see the comparison of how the additivity of indices is changed depending on the type of effects present in a function. For functions with both interaction and inflation effects, we have three possible scenarios where either one of the effects is dominant, or both effects are equal in scale and cancel out. This means that regardless of which effect is dominant, we cannot accurately quantify the true scale of the effect due to the presence of the other effect.

However, we can also detect inflation effects through a comparison of the naive bounds of a variable against the enclosure of the subintervalised outputs. This gives us an alternate method of detecting variables with inflation effects when interaction effects are also present in the function, allowing us to have awareness that there may be a hidden effect in the function.

Table 2. Summary table showing the relationship between the sum of the individual indices and the simultaneous index within subinterval sensitivity for all three possible function types depending on whether interaction effects are present within the function.

| Expression | No Interactions | Interactions |
|---|---|---|
| Single-Use or Repeated without Inflation | $S_i + S_j = S_{ij}$ | $S_i + S_j > S_{ij}$ |
| Repeated with Inflation | $S_i + S_j < S_{ij}$ | $S_i + S_j \lesseqgtr S_{ij}$ |

## 8. Conclusion

In this paper, simultaneous indices are introduced into subinterval sensitivity through subintervalising multiple variables at the same time. This extension of subinterval sensitivity allows for the comparison of sensitivity indices of pairs, triplets, and other high-order groupings of variables against the sum of the individual indices of all constituent variables. Through this comparison, we can detect and quantify interaction effects within subinterval sensitivity. For variable interactions, the sum of individual indices is higher than the respective simultaneous index. In contrast, for expressions with repeated variables causing inflation, the sum of individual indices is lower than the respective simultaneous index.

By knowing which variables have interaction effects, we can more accurately state which variables require attention and which variables can be ignored or changed freely within a model. As the detection of interaction effects can happen even when a small number of subintervals are used, subinterval sensitivity can be used to cheaply and rigorously determine if interaction effects exist. This usage of subinterval sensitivity is enabled by the rigor of the method, where the indices obtained are highly reliable even in low-data applications, which can occur where gathering data is expensive or time-consuming. In contrast, variability due to sampling can be a major factor that can affect the precision and rigor of sampling-based methods including variance-based indices.

We also introduce a method for detecting inflation indices through the comparison of the naive bounds and the enclosure of the subinterval outputs for each variable in both individual and simultaneous subintervalisation. With the inflation detection tool, we can determine if a dependency problem exists within a function, which allows us to determine if using subinterval sensitivity has any risk of obtaining inaccurate results. Detecting inflation is particularly useful for complex functions or black-box models where visually identifying repetition may be difficult or not possible.

The presence of both interaction and inflation effects in the same function results in two distinct and opposite effects within subinterval sensitivity. This counteracting behavior is visible in Fig. 7, where the introduction of inflation effects by re-writing a function changes the sensitivity indices obtained from the function. Having both interaction and inflation effects present in a function introduces artefactual uncertainty, which results in a scenario where the accuracy of the indices obtained can no longer be guaranteed.

As inflation effects are unique to subinterval sensitivity, these findings highlight the need for methods within interval arithmetic to mitigate and eliminate inflation. These methods would be especially important for black-box models, where the model is unknown, or when the model's expression cannot be re-arranged in single use. The presence of inflation effects also limits the reliable application of the method in real-world scenarios to functions without repetition or where repetition does not cause inflation.

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## Reproducibility

The code utilized to produce the results used within this paper
is accessible in this repository: `https://github.com/dawidochnio/Interaction_effects_in_subinterval_sensitivity_analysis`

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