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A Weibull Reliability Qualification Test Plan Derivation Method based on Similar Product

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Reliability Qualification Test (RQT) is widely exercised in engineering, to decide whether product reliability meets specified requirement. RQT is usually mandatorily carried out by referring to the test plans from standards GJB899A(in China) or MIL-STD-781(in U.S.). Most applicable RQTs are Type-I censored, due to the ease of arrangement. The Type-I censored test plans in the standards, often require long test time, and those with short test time attach high producer's and customer's risks. Besides, the usage of the standards assumes that product lifetime follows exponential distribution. For Weibull distributed products, which are also popular in practice, there were few research on deriving RQT plans. To cope with above problems, taking into account the prior information from similar products, which is usually in terms of reliability at certain time, this paper proposes an RQT plan derivation method, to calculate the producer's and customer's risks, and derive the suitable test plans subjected to the risks restraints. A numerical example is presented to illustrate that the test plans derived by the proposed method have much shorter test time and lower risks than that in standard GJB899A. And the robustness of the method is validated in the sensitivity analysis.

Keywords: Reliability Qualification Test, Type-I Censoring, Similar product reliability, Producer's risk, Customer's risk.

1. Introduction

Reliability qualification test (RQT), sometimes also called reliability demonstration test (RDT), or reliability acceptance test (RAT), is widely used in product development to qualify whether a batch of products meet predetermined reliability requirements, which are mainly described in terms of reliability indices, for example, MTBF (Mean Time between Failures) and failure rate (Lee, Makam, McKelvey & Lu, 2015). RQT should be carefully planned in advance, as a batch of products will be accepted or rejected based on the data of the test samples. There exists two risks in the test, producer's risk α and consumer's risk β, which should be negotiated and approved by the producer and consumer. α is the risk that the products are rejected given that their reliability meets the requirements, and β is the risk that the products are accepted given that their reliability does not meet the requirements. Along with the two risks, θ_0 and θ_1, should be arranged when planning an RQT. θ_0 and θ_1 are the acceptable and the reject values for the reliability index θ, respectively. So RQT plans are derived to meet the predetermined parameters (θ_0, θ_1, α, β).

RQTs are performed on the samples and life data are collected, based on which decision is made whether a batch of products meet reliability requirements. Type-I censoring (also called time censoring) RQT is most widely applied due to the easy manipulation of its test plan (T, C), where T is the cumulative test time and C is the maximum number of allowable failures to pass the test. Type-I censoring mechanism means that the RQT will be terminated when the cumulative test time (by summating operating times of all the samples) reaches T, and the number of failures c is recorded. This batch of products will be accepted if c⩽C, and will be rejected otherwise.

Standards such as the Chinese GJB899A-2009 (Standard, 2009) and the US Mil-Hdbk-781D (Standard, 1987), provide guidance for implementation of RQT. Given test parameters (θ_0, θ_1, α, β), feasible RQT plan (T, C) can be found in these standards. However, the applying such standard assumes that the product lifetime follows an exponential distribution, which is not usually satisfied.

There is also extensive literature on deriving RQT plans under certain conditions. Type-I censoring RQTs usually suppose that the lifetimes of the products follow certain probability distributions. For example, binomial distributed products are first and widely considered in RQT practices (Mease and Nair, 2006)(Yuan and Li, 2014) (Li et al., 2017). Exponential distribution is most widely assumed in RQT plan derivation (Fitzgerald, Martz and Parker, 1999) (Xu et al., 2017). As Weibull distribution is widely used in reliability practices, some research has also studied RQT for Weibull distributed products(Sun and Berger, 1994)(Guo and Liao, 2012) (Jang, Ahn, etc, 2013) (Zhu, 2020). However, most of the researches on Weibull products assumed that the shape parameter is known.

Most of RQTs are conducted at use conditions, which may require long test time for highly reliable products. Consequently, accelerated test attracts more and more attentions as it can significantly reduce test time and cost (Ren et al., 2014; Li et al., 2017; Kim et al., 2019; Wilson and Farrow 2021).

For statistically designed reliability acceptance tests, such as Bayesian testing and Bogey testing, to qualify products following Weibull distribution, the desired test time is far beyond the expected reliable life (Dodson and Klerx, 2012). One option is taking some other information into consideration, for example, reliability growth data during development (Zhang et al., 2024), experts judgments (Tan et al., 2025), subsystem data (Jiang et al., 2022) and so on. As long as Weibull distributed products are concerned, no literature exists in which similar product reliability is fused to develop RQT plans.

In this paper, an RQT plan derivation method is developed for Weibull distributed products, by fusing the reliability estimate from similar product. The rest of the paper is organized as follows: Section 2 introduces the Weibull model. Similar product reliability is formulized in Section 3. Section 4 proposes the method to derive RQT plans that combines similar product data. Section 5 provides sensitivity analysis on our proposed method. It is summarized in Section 6.

2. Formulation of the RQT

We will first introduce Weibull model, as in this paper, product lifetime is following Weibull distribution. Then the RQT problem is formulated.

2.1. *Construction of Weibull model*

Weibull distribution is probably the most widely used lifetime distribution, due to its versatility. The cumulative distribution function (CDF) of Weibull distribution is

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^m\right], \tag{1}$$

where η and m are the scale and shape parameters, respectively and are both undetermined. Let $\lambda = 1/\eta^m$, Eq.(1) can be expressed by

$$R(t) = \exp(-\lambda t^m). \tag{2}$$

Let a random variable $u = t^m$. Then u is following exponential distribution. According to the properties of exponential distribution, in the Type-I censoring, the probability that r failures occur during time interval [0, T] can be described by

$$P(r) = \frac{(\lambda u_0)^r}{r!} e^{-\lambda u_0}, \tag{3}$$

where $u_0 = T^m$ is also a variable as m is not determined.

According to above introduction, the shape parameter m is considered as equally distributed within the interval [m_1, m_2], so it can be described by a uniform distribution over [m_1, m_2]. Its pdf is

$$\pi(m) = \frac{1}{m_2 - m_1}, \; m \in [m_1, m_2] \cdot \tag{4}$$

Based on the distribution of shape parameter m, we will derive the distribution of the scale parameter λ, which fulfills the objective of deriving the joint pdf, $\pi(\lambda, m)$, of the two parameters λ and m. According to Zhao et al. 2021, product reliability R_τ at time τ is following Negative log-gamma distribution with pdf as

$$\pi(R_\tau) = \frac{b^a}{\Gamma(a)} (R_\tau)^{b-1} (-\ln R_\tau)^{a-1}, \text{ for } 0 < R_\tau < 1 \tag{5}$$

where a and b are two parameters for the distribution. Take the derivative of Eq.(2) with respect to λ, we have

$$\frac{dR_\tau}{d\lambda} = -\tau^m \exp(-\lambda \tau^m) < 0, \tag{6}$$

where R_τ decreases when λ increases. So the following Eq.(7) holds.

$$\begin{aligned} P(\lambda < x) &= P(R_\tau > \exp(-x\tau^m)) \\ &= \int_{\exp(-x\tau^m)}^{1} \pi(R_\tau) dR_\tau \end{aligned} . \tag{7}$$

Take the derivative of $P(\lambda < x)$ with respect to x, we have

$$f_\lambda(x) = \frac{(b\tau^m)^a}{\Gamma(a)} x^{a-1} \exp(-b\tau^m x), \tag{8}$$

which is the pdf of scale parameter λ, and can be further expressed by

$$\pi(\lambda \mid m) = \frac{(b\tau^m)^a}{\Gamma(a)} \lambda^{a-1} \exp(-b\tau^m \lambda). \tag{9}$$

Then the joint pdf of the two parameters λ and m is

$$\pi(\lambda, m) = \frac{1}{m_2 - m_1} \frac{(b\tau^m)^a}{\Gamma(a)} \lambda^{a-1} \exp(-b\tau^m \lambda), \tag{10}$$

where $\lambda \in [0, \infty]$, $m \in [m_1, m_2]$. The hyperparameters a and b will be determined by prior information, which is introduced in the next section.

2.2 *Formulation of the RQT*

In this paper, as type-I censored RQT is taken into account, reliability lifetime t_q, is chosen as the acceptance index. According to Eq.(2), given reliability q, reliability lifetime t_q can be derived by

$$t_q = \left[-\ln(q) / \lambda\right]^{1/m}.$$

In an RQT, the decision whether to accept a batch of products is made by carrying out the following hypothesis test, restricted to predetermined producer's risk α_0 and consumer's risk β_0,

$$H_0: \; t_q > \theta_0, \;\; H_1: t_q < \theta_1, \tag{11}$$

where θ_0 is the acceptable value for the reliability lifetime t_q, and θ_1 is the reject value for t_q, usually, $\theta_0 > \theta_1$. To formulize the problem, we have following assumptions:

(1) The lifetime of product follows a Weibull distribution with shape parameter falling within the interval [m_1, m_2], which is determined by expertise or similar products.

(2) The RQT is Type-I censoring with replacements.

(3) Similar product reliability on the product reliability is introduced as prior information, which is expressed by $R_0(\tau_0)$, means product reliability at time τ_0.

3. Use of Similar Product Reliability

As mentioned above, similar product reliability is used as extra information, to calculate the hyperparameters a and b in Eq.(10). The similarity measurement between current product with similar product is not necessary in this paper, as the calculation results are slightly affected by the variations of estimates of similar product reliability, which will be further illustrated in subsequent sensitivity analysis.

The traditional max-entropy method is applied to calculate the hyperparameters by similar product reliability, to obtain the pdf $\pi(\lambda, m)$.

As similar product reliability is given in the form of $R_0(\tau_0)$, it can be further expressed as

$$\int_0^1 R_{\tau_0}\pi(R_{\tau_0})dR_{\tau_0} = R_0 . \quad (12)$$

Then max-entropy method is adopted here, and a nonlinear programming is constructed by

$$\begin{cases} \max H = -\int_0^1 \pi(R_{\tau_0}) \ln\left[\pi(R_{\tau_0})\right] dR_{\tau} \\ s.t.\ \int_0^1 R_{\tau_0}\pi(R_{\tau_0})dR_{\tau_0} = R_0 \end{cases}, \quad (13)$$

which can be simplified as

$$\begin{cases} \max H = -a\ln b + \ln\Gamma(a) + A - BD \\ s.t.\ \left(\dfrac{b}{b+1}\right)^a = R_0 \end{cases}, \quad (14)$$

where $A = a(b-1)/b$, $B = \int_0^1 (R_{\tau_0})^{b-1}(-\ln R_{\tau_0})^{a-1}\ln[-\ln(R_{\tau_0})]dR_{\tau_0}$, $D = (a-1)b^a/\Gamma(a)$. And the two hyperparameters a and b have the relationship of

$$a = \ln(R_0)/(\ln(b) - \ln(b+1)) . \quad (15)$$

So Eq.(14) is transformed into a one variable nonlinear programming. The two hyperparameters a and b are derived by solving the nonlinear programming (14). Then the joint pdf $\pi(\lambda, m)$ is obtained.

4. Derivation of RQT Plans

The similar product reliability are used to derive RQT plans. To achieve this goal, we should first propose the two risks calculation method, which makes use of similar product reliability. Then the RQT plan derivation method is introduced subjected to the risks constraints.

4.1 *Calculating the two risks*

4.1.1. *Producer's risk calculation*

As stated, producer's risk α is the probability that products are rejected while the reliability lifetime t_q is greater than θ_0. So α can be calculated as

$$\begin{aligned} 1-\alpha &= \frac{\sum_{r=0}^{C} P(r, t_q > \theta_0)}{P(t_q > \theta_0)} \\ &= \frac{\sum_{r=0}^{C}\int_{m_1}^{m_2}\int_0^{\theta_0^{-m}(-\ln q)} P(r)\pi(\lambda, m)d\lambda dm}{\int_{m_1}^{m_2}\int_0^{\theta_0^{-m}(-\ln q)} \pi(\lambda, m)d\lambda dm} . \end{aligned} \quad (16)$$

Substitute Eqs.(3) and (10) into Eq.(16), we have

$$\alpha = 1 - \frac{\sum_{r=0}^{C}\int_{m_1}^{m_2}\int_0^{\theta_0^{-m}(-\ln q)} \dfrac{(\lambda u_0)^r}{r!} e^{-\lambda u_0} \dfrac{1}{m_2 - m_1} \dfrac{(b\tau^m)^a}{\Gamma(a)} \lambda^{a-1}\exp(-b\tau^m\lambda)d\lambda dm}{\int_{m_1}^{m_2}\int_0^{\theta_0^{-m}(-\ln q)} \dfrac{1}{m_2 - m_1} \dfrac{(b\tau^m)^a}{\Gamma(a)} \lambda^{a-1}\exp(-b\tau^m\lambda)d\lambda dm} . \quad (17)$$

4.1.2. *Customer's risk calculation*

Customer's risk β is the probability that products are accepted while the reliability lifetime t_q is less than θ_1. It can be calculated in the similar way as

$$\beta = \frac{\sum_{r=0}^{C} P(r, t_q < \theta_1)}{P(t_q < \theta_1)} = \frac{\sum_{r=0}^{C} \int_{m_1}^{m_2} \int_{\theta_1^{-m}(-\ln q)}^{\infty} \frac{(\lambda u_0)^r}{r!} e^{-\lambda u_0} \frac{1}{m_2 - m_1} \frac{(b\tau^m)^a}{\Gamma(a)} \lambda^{a-1} \exp(-b\tau^m \lambda) d\lambda dm}{\int_{m_1}^{m_2} \int_{\theta_1^{-m}(-\ln q)}^{\infty} \frac{1}{m_2 - m_1} \frac{(b\tau^m)^a}{\Gamma(a)} \lambda^{a-1} \exp(-b\tau^m \lambda) d\lambda dm}. \quad (18)$$

4.2 *Deriving test plans*

RQT plan (T, C) is derived by setting initial values for the two parameters T and C, and gradually increasing the values while keep the two risks under control. The procedure is as below.

(i) Determine the acceptance value θ_0, reject value θ_1, the thresholds of producer's risk α_0, customer's risk β_0, the concerned operation time τ_0, and the maximum allowable failures C_0;

(ii) Similar product reliability is given in the form of $R_0(\tau_0)$, then $\pi(\lambda, m)$ is obtained by solving Eqs.(14) and (15);

(iii) Set initial values for T and C, for example, T=100 and C=1;

(iv) Calculate the actual producer's risk α by Eq.(17);

(v) Check whether $\alpha \le \alpha_0$ holds: If $\alpha > \alpha_0$, let T=T+10 and go to step (4); If $\alpha \le \alpha_0$, go to step (vi);

(vi) Calculate the actual customer's risk β by Eq.(18);

(vii) Check whether $\beta \le \beta_0$ holds: If $\beta > \beta_0$, let C=C+1 and go to step (4); If $\beta \le \beta_0$, go to step (8);

(viii) Output the test plan (T, C), and go to step (ix);

(ix) Check whether $C < C_0$ holds: If it holds, there are alternative test plans, then let T=100, and C=C+1, turn to step (4); If no, terminate the search.

5. A Numerical Example

To validate the proposed RQT plan derivation method, a numerical example is applied here.

Assume that in this case:

(i) The product to be qualified has a lifetime following Weibull distribution with parameters η=900, $m_1 = 1$, $m_2 = 3$, which means $m \in [1, 3]$.

(ii) The acceptable value for the reliability lifetime t_q is θ_0=820h, and the reject value for t_q is θ_1 =410h, where q=0.9. The producer's risk α and customer's risk β are both restricted to 0.2, namely, $\alpha_0 = \beta_0$ =0.2. The operation time τ_0=100. And the maximum number of allowable failures C_0=0,1.

(iii) The similar product reliability is given in the form of $R_0(\tau_0)$=0.9636 ,where τ_0=100.

According to above assumptions, we will illustrate how to derive the time-censored RQT plans.

As similar product reliability is given as $R_0(100)=0.9636$, according to Section 3, we can solve for the hyperparameters a and b of $\pi(\lambda,m)$. By nonlinear programming, b=0.0253 is first found which maximizes the entropy. Then a=0.01 is solved by referring to Eq.(15).

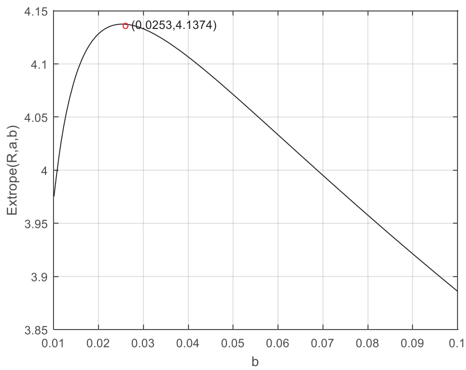

Fig.1 Solving for b

Substitute a and b into Eq.(10), $\pi(\lambda,m)$ is determined. Then following the procedures in Section 4.2, producer's and customer's risks over test time can be calculated and are plotted in Fig.2, when the maximum number of allowable failures C_0=0, i.e., no failures are permitted during the test. From Fig.2, we can see that the customer's risk decreases dramatically over test time, while producer's risk increases slightly over test time.

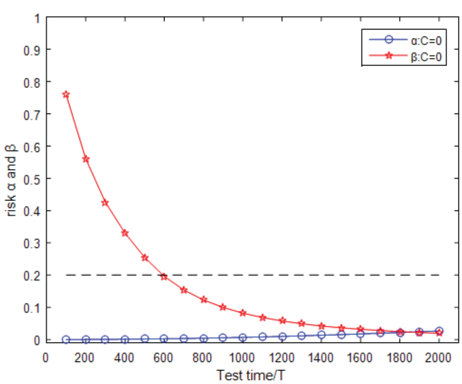

Fig.2 Risks over test time(C_0=0)

If we set C_0=1, and plot the risks over test time in the same Fig.2 as C_0=0.

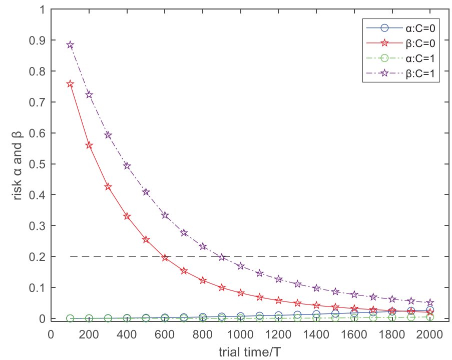

Fig.3 Risks over test time(C=0 and 1)

From Fig 3, we can see the when the maximum number of allowable failures C_0 changes from 0 to 1, producer's risk decreases while customer's risk increases, compared at any given test time. This is coordinating with our experience as increasing of C_0 will lead to higher risk for customer, and lower risk for producer at the same time.

Then we can find the feasible test plans($\alpha \le \alpha_0$ and $\beta \le \beta_0$) from Fig.3 and list them in Table.1 below. We only choose the test plans with test time≤1500. The last row is the test plan from GJB899A, matching the predefined conditions.

Table.1 Test plans (T,C) derived by our method and GJB899A($\alpha_0=\beta_0$=0.2)

Test plan No.		Test time(T)	C	α	β
Test plans derived by our method	1	600	0	0.002604	0.195996
	2	700	0	0.003486	0.153784
	3	800	0	0.00451	0.122844
	4	900	0	0.005677	0.099736
	5	1000	0	0.00699	0.082161
	6	1100	0	0.008446	0.068459
	7	1200	0	0.010042	0.057736
	8	1300	0	0.011774	0.049168
	9	1400	0	0.013634	0.042254
	10	1500	0	0.015614	0.036618
Test plan from GJB899A	1*	3198	9	0.199	0.21

From Table.1, it is clear that all the test plans (No.1,…,10), derived by our proposed method, are feasible according to our predefined conditions, and are all superior to the test plan 1* from GJB899A, in terms of test time or the two risks α and β. So our proposed method, by integrating similar product reliability into the risk calculation, has the advantages of shorter test time and smaller risks, compared with the test plan in GJB899A.

6. Conclusion

RQT is widely used to qualify whether a batch of products meet predetermined reliability requirements. RQT is usually mandatorily carried out by referring to the test plans in standard GJB899A(in China) or MIL-STD-781(in U.S.). Either standard requires that product lifetime follows exponential distribution and those test plans with short test time have high producer's risk α and consumer's risk β. If the lifetime of a product follows some distribution other than exponential, or short test time with low risks is desirable, the standards are no longer applicable. To cope with the problem, when product's lifetime follows Weibull distribution, which is also very common in engineering cases, this paper presents a novel method to derive its RQT plans, by integrating similar product reliability on the product reliability at given time τ_0. A numerical example exhibits the advantage of our proposed method on the derived test plans with shorter test time as well as low risks, compared with that in GJB899A.

Although the proposed procedures to derive test plans in Section 4.2 can not ensure they are exact optimal, in term of minimum test time, due to the fixed search step, as the time is gradually increasing, the derived test plans should be close to the optimal one. Meanwhile, we are investigating intelligent search algorithms, such as heuristic algorithm, to optimize the search method. A calculation program is under developing to find the optimal test plan conveniently.

Further researches are to be carried out on different lifetime distributions and fusing multi-source information other than similar product reliability, such as subsystem test data, history data, or data from similar product.

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