Proc. of the European Safety and Reliability Conference (ESREL 2026)
Edited by José C. Matos, Paulo B. Lourenço, Michael Beer, Daniel Oliveira, Edoardo Patelli, Hélder S. Sousa, Rui A. Silva and Monica Santamaria

doi: 10.3850/ESREL2026061419_esrel26-p26385-cd

A bounded Transformed Wiener process with change-points with an application to lithium-ion batteries degradation data

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Many technological units are subjected during their operating life to a degradation process that, in the long run, leads to their failure. Usually, a failure is assumed to occur when their degradation level exceeds an assigned failure threshold. The capability to accurately describe and predict the temporal evolution of the degradation of these units is critical for remaining useful life prediction and maintenance planning. The existing literature gives evidence that, frequently, the degradation process of these units exhibits change-points where the degradation rate or the mechanism of degradation changes abruptly, e.g. due to altered material properties or to the action of unobservable external factors. The diagnostic and prognostic accuracy of the model adopted to describe these degradation phenomena highly depends on their capability to account for the presence of these change-points.
In this paper, we propose some degradation models with change-points that describe the whole degradation phenomenon via bounded Transformed Wiener processes where the functional form of the drift function and other features of the model can be phase-specific. The main properties of the model are illustrated. The distribution of the degradation level at a given time and the conditional distribution of the future degradation increment are formulated. The estimation of its parameters, including the change-points location, is performed by maximum likelihood. An application to a real set of degradation data of lithium-ion batteries is presented, which proves the utility of the proposed model and the feasibility of the adopted estimation procedure.

Keywords: Wiener process, change-points, bounded degradation phenomena, maximum likelihood estimation.

1. Introduction

Degradation model-based approaches have been proven to be very effective for reliability assessment of degrading technological units. The link between degradation and reliability is usually established by assuming that these units fail when their degradation level passes an assigned failure threshold. Especially in the case of units that are highly reliable, this technique allows to overcome the limitations determined by the lack of failure data. In fact, by using degradation models it is possible to estimate the reliability even in the absence of failure data, by using (less costly) degradation measurements.

Furtherly, the use of degradation models allows to create a link between the degradation state of the unit and its remaining useful life, which is the key element for planning condition-based maintenance activities.

Obviously, a pivotal aspect for the success of this approach, is the accurate description of the degradation process of the unit of interest. Models like the gamma process (Abdel-Hameed 1975), the inverse Gaussian process (Wang and Xu 2010), and the Wiener process (Whitmore and Schenkelberg 2024), have demonstrated to be suitable to describe many real world degradation phenomena. However, recent literature has shown that many real sets of degradation data give evidence of the existence of multiple phases of the underlying degradation phenomena, which cannot be described by using the same model, or to say better, by only one of these classical degradation models, in their basic form. Indeed, the existence of these phases can be due to different causes, such as (for example) incomplete burn-in, changes of the physical degradation mechanism, and/or (unobservable) changes in the external working conditions.

In these cases, to describe the whole degradation process, instead of using a single, complex, degradation model, many authors suggest to adopt a different (simpler) model in each phase. Instants of time that separate successive phases are referred to as change-points and the overall model is referred to as degradation process with change-points.

In the literature there are many examples of degradation models with two or more change-points that have been developed by using as reference model the classical gamma process (Ling et al. 2019), (Lin et al. 2021), the Inverse Gaussian process (Zhuang et al. 2024), (Qiao et al. 2025), and the Wiener process (Yan et al. 2016), (Zhang et al. 2019).

In this paper we propose some degradation models with change-points developed by using as reference models some new bounded characterizations of the transformed Wiener process (Giorgio and Pulcini 2018). The proposed models are suitable for describing non-monotonically increasing degradation phenomena that are intrinsically bounded above, for example, those in which the degradation level is expressed as a percentage reduction of a given health index.

To the best of our knowledge, the proposed models are the first examples of bounded processes with change-points.

The manuscript is structured as it follows. In Section 2 we illustrate the main properties of the proposed models and formulate the conditional distribution of the future degradation increment. In Section 3 we address the maximum likelihood estimation of their parameters, which include the change-points location. Section 4 presents a numerical example, developed by using a real set of degradation data of lithium-ion batteries, which gives evidence of the utility of the proposed model and demonstrates the affordability of the adopted one-step estimation procedure. Section 5 is devoted to conclusions.

2. Bounded Transformed Wiener Process with change-points

We assume that the degradation model in each given phase is modeled by a bounded transformed Wiener process (BTWP), say $\{W(t), t \geq 0\}$. Being Markovian, with age and state dependent increments, given an initial condition (here $W(0) = 0$), the BTWP is fully defined by the conditional cumulative distribution function (Cdf) of its degradation increment $\Delta W(t, t + \Delta t) = W(t + \Delta t) - W(t)$ over the (generic) time interval $(t, t + \Delta t)$, given the current degradation level $W(t) = w_t$. This conditional Cdf can be expressed as:

$$F_{\Delta W(t,t+\Delta t)|W(t)}(\delta|w_t) = \Phi\left[\frac{\Delta g(w_t, w_t + \delta) - \mu\,\Delta\Lambda_\mu(t, t + \Delta t)}{\sigma\sqrt{\Delta\Lambda_\sigma(t, t + \Delta t)}}\right], \quad (1)$$

where μ and σ are two positive parameters, called drift and diffusion coefficients, respectively, $\Delta\Lambda_\mu(t, t + \Delta t) = \Lambda_\mu(t + \Delta t) - \Lambda_\mu(t)$, $\Lambda_\mu(t)$ is the drift function, $\Delta\Lambda_\sigma(t, t + \Delta t) = \Lambda_\sigma(t + \Delta t) - \Lambda_\sigma(t)$, $\Lambda_\sigma(t)$ is the monotonic non-decreasing diffusion function, $\Delta g(w_t, w_t + \delta) = g(w_t + \delta) - g(w_t)$, and $g(w)$ is a monotonically increasing, differentiable, invertible function defined in $\mathbb{R}$ that takes value in $(-\infty, U_w)$, where U_w is the upper bound of $\{W(t), t \geq 0\}$.

In this paper we model $g(w)$ as:

$$g(w) = \begin{cases} w \quad , & w \leq 0 \\ -U_w \ln\left(1 - \dfrac{w}{U_w}\right), & 0 < w < U_w \end{cases} \quad (2)$$

and assume that U_w is known.

However, if deemed appropriate, even the upper bound can be treated as an unknown parameter, as in (Fouladirad et al. 2023), where the proposed model is a bounded version of the transformed gamma process (Giorgio et al., 2015).

From Eq. (1), the conditional probability density function (pdf) of the degradation increment $\Delta W(t, t + \Delta t)$, given $W(t) = w_t$, can be expressed as:

$$f_{\Delta W(t,t+\Delta t)|W(t)}(\delta|w_t) = \frac{g'(w_t+\delta)e^{-\frac{1}{2}\left(\frac{\Delta g(w_t,w_t+\delta)-\mu\,\Delta\Lambda_\mu(t,t+\Delta t)}{\sigma\sqrt{\Delta\Lambda_\sigma(t,t+\Delta t)}}\right)^2}}{\sqrt{2\pi}\sigma\sqrt{\Delta\Lambda_\sigma(t,t+\Delta t)}}, \quad (3)$$

where $g'(w_t+\delta)$ is the first derivative of the state function $g(w_t+\delta)$ with respect to δ:

$$g'(w_t+\delta) = \begin{cases} 1 \quad , & w_t+\delta \le 0 \\ \frac{U_w}{U_w - w_t - \delta}, & 0 < w_t+\delta < U_w \end{cases}. \quad (4)$$

From (1) and (3), replacing w_t and t by 0, δ by w, and Δt by t, the Cdf and pdf of the degradation level $W(t)$ at time t are given by

$$F_{W(t)}(w) = \Phi\left[\frac{g(w)-\mu\,\Lambda_\mu(t)}{\sigma\sqrt{\Lambda_\sigma(t)}}\right] \quad (5)$$

and

$$f_{W(t)}(w) = \frac{g'(w)}{\sqrt{2\pi}\,\sigma\sqrt{\Lambda_\sigma(t)}} e^{-\frac{1}{2}\left(\frac{g(w)-\mu\,\Lambda_\mu(t)}{\sigma\sqrt{\Lambda_\sigma(t)}}\right)^2}, (6)$$

where $g'(w)$ is the first derivative of $g(w)$ with respect to w:

$$g'(w) = \begin{cases} 1 \quad , & w \le 0 \\ \frac{U_w}{U_w - w}, & 0 < w < U_w \end{cases}. \quad (7)$$

The mean and variance of $W(t)$, as well as the conditional mean and variance of the increment $\Delta W(t, t+\Delta t)$, can be computed via numerical integration.

In this work, we propose two models: one with a single change-point and one with two change-points.

Under the first BTWP, the drift function $\Lambda_\mu(t)$ is assumed to be phase dependent and is modelled as:

$$\Lambda_\mu(t) = \begin{cases} e^{t/b} - 1 \quad , & t \le t_c \\ e^{t/b} - 1 + c\left(e^{\frac{t-t_c}{d}} - 1\right), & t \ge t_c \end{cases}, \quad (8)$$

where $b, c, d > 0$ and t_c is the unknown (single) change-point.

The drift coefficient μ, the diffusion coefficient σ, and the diffusion function $\Lambda_\sigma(t)$ are assumed to be phase independent (i.e., they are supposed to assume the same values/functional form in all the phases).

For the diffusion function we have considered two possible characterizations, say:

$$\Lambda_\sigma(t) = e^{t/a} - 1\,, \quad a > 0 \quad (9)$$

$$\Lambda_\sigma(t) = t^a\,, \quad a > 0. \quad (10)$$

Under the second BTWP (i.e., the one with two change-points), the drift function $\Lambda_\mu(t)$, that is still phase specific, is modelled as:

$$\Lambda_\mu(t) = \begin{cases} 0 \quad , & t \le t_{c1} \\ e^{\frac{t-t_{c1}}{b}} - 1 \quad , & t_{c1} < t \le t_{c2} \\ e^{\frac{t-t_{c1}}{b}} - 1 + c\left(e^{\frac{t-t_{c2}}{d}} - 1\right), & t > t_{c2} \end{cases}, \quad (11)$$

where b, c, and d are three positive parameters, and t_{c1} and $t_{c2} \ge t_{c1} \ge 0$ are the unknown change-points.

As in the case of the BTWP with one change-point, the drift coefficient μ is assumed to take the same value in all the phases.

The functional form of the diffusion function $\Lambda_\sigma(t)$ is assumed again to be the same in all the phases.

However, for the second model we use for $\Lambda_\sigma(t)$ the following three alternative expressions:

$$\Lambda_\sigma(t) = e^{t/a} - 1\,, \quad a > 0\,, \quad (12)$$

$$\Lambda_\sigma(t) = t^a\,, \quad a > 0\,, \quad (13)$$

$$\Lambda_\sigma(t) = t\,, \quad (14)$$

and assumed that the diffusion coefficient σ is phase independent.

Furthermore, as an alternative, only in the case of the model obtained by using the diffusion function (14), we assume that σ can take the value σ_0 up to t_{c2} and the value σ_2 after t_{c2}:

$$\sigma(t) = \begin{cases} \sigma_0 & t \le t_{c2} \\ \sigma_2 & t > t_{c2} \end{cases}. \quad (15)$$

In this case, the product $\sigma^2(t)\Lambda_\sigma(t)$ is given by:

$$\sigma^2(t)\Lambda_\sigma(t) = \begin{cases} {\sigma_0}^2 t \quad , & t \le t_{c2} \\ {\sigma_0}^2 t_{c2} + {\sigma_2}^2(t - t_{c2})\,, & t > t_{c2} \end{cases}. \quad (16)$$

3. Maximum Likelihood Estimation

Let us suppose that m units, operating under identical conditions, are subject to point inspections which allow to reveal their degradation level. In particular, let us indicate by n_i $(i = 1, \ldots, m)$ the number of inspections performed on the unit i, by $t_{i,j}$ ($j = 1, \ldots, n_i$) the time of the $j-$th inspection performed on the unit i, and by $w_{i,j}$ the value of the degradation level $W(t_{i,j})$ of the unit i at $t_{i,j}$. Then, under the proposed BTWPs with change-points, the likelihood function relative to these data can be formulated as:

$$\mathcal{L}(\boldsymbol{\xi}, \boldsymbol{w}) = \prod_{i=1}^{m} \prod_{j=1}^{n} f_{\Delta W(t_{i,j-1}, t_{i,j}) | W(t_{i,j-1})} (\delta_{i,j} | w_{i,j}) \quad (17)$$

where, $\boldsymbol{w} = \{w_{1,1}, \ldots, w_{i,j}, \ldots w_{m,n_m}\}$ is the set of available degradation data, $\boldsymbol{\xi}$ is the vector of model parameters, $\delta_{i,j} = w_{i,j} - w_{i,j-1}$, and $w_{i,0} = t_{i,0} = 0 \ \forall i$.

More specifically, under the first model (the one with one change-point) it is $\boldsymbol{\xi} = \{\mu, \sigma, a, b, c, d, t_c\}$, while under the second BTWP model (the one with two change-points) it is:

(i) $\boldsymbol{\xi} = \{\mu, \sigma, a, b, c, d, t_{c1}, t_{c2}\}$, when the function $\Lambda_\sigma(t)$ is modelled as in Eq. (12) and Eq. (13), and σ takes the same value in all the phases,

(ii) $\boldsymbol{\xi} = \{\mu, \sigma_0, \sigma_2, b, c, d, t_{c1}, t_{c2}\}$, when $\Lambda_\sigma(t)$ is modelled as in Eq. (14) and σ is given by Eq. (15).

Under both models, the upper bound U_w is here assumed to be known. However, if U_w is unknown, it can be easily assumed to be a parameter to be estimated.

The maximum likelihood (ML) estimate of $\boldsymbol{\xi}$ is the vector value $\hat{\boldsymbol{\xi}}$ that maximizes the likelihood function over the parameter space $\boldsymbol{\Omega}_{\xi}$:

$$\hat{\boldsymbol{\xi}} = \underset{\xi \in \Omega_\xi}{\operatorname{argmax}} \, \mathcal{L}(\boldsymbol{\xi}, \boldsymbol{w}). \quad (18)$$

Thus, for example, in the case of the first model where $\boldsymbol{\xi} = \{\mu, \sigma, a, b, c, d, t_c\}$ it is $\boldsymbol{\Omega}_{\xi} = (0, \infty)^7$.

In this paper we have determined $\hat{\boldsymbol{\xi}}$ by using a numerical optimization procedure. The convergence of the optimization procedure to a global maximum (i.e., to the maximum likelihood estimates) was ensured by. The convergence of the maximization algorithm was verified using multiple starting points. The ML estimate of any function of model parameters can be readily obtained (exploiting the invariance property of ML estimators) by computing the function at the ML estimates of the parameters it depends on.

4. Numerical example

In order to show the flexibility of the proposed degradation models and the feasibility of the adopted estimation approach, a demonstrative application was developed on the basis of a real set of measurements of capacity of four lithium-ion battery cells obtained via lab experiments, displayed in Figure 5 of (Wang et al. 2023).

The capacity of the cells is expressed in ampere-hours (Ah), and the operating time is expressed in number of cycles. In our application, we have used as degradation measurements the percentage loss of capacity, where the loss of each cell is evaluated with respect to the capacity value measured at the first cycle.

The whole data set consists of 118 measurements (29 for the unit 1, 29 for the units 2, 28 for the unit 3, and 32 for the units 4), so that the total number of observed capacity losses is equal to 114. The data set containing the percentage loss of capacity of the four battery cells is reported in Table 1.

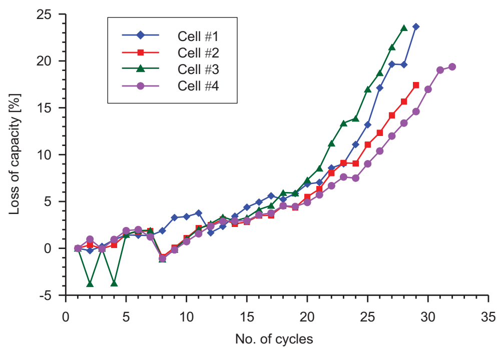

Fig. 1. Percentage loss of capacity of the four lithium-ion battery cells.

Figure 1 gives a visual representation of the degradation path of the four cells. From this figure it is apparent that the considered degradation phenomenon may present change-points. In addition, the same paths show the presence of negative increments (see also Table 1), that prevent the use of monotonically increasing degradation processes.

Based on these motivations, we decided to apply the proposed BTWPs to these data.

In the application, due to the intrinsic nature of the considered degradation data, we assumed that the upper bound U_w of the degradation

Table 1. Degradation data: percentage loss of capacity as a function of number of cycles of the four lithium-ion battery cells, from (Wang et al. 2023).

No. of cycles	Cell #1	Cell #2	Cell #3	Cell #4
1	0.00	0.00	0.00	0.00
2	– 0.24	0.37	– 3.77	0.98
3	0.21	– 0.03	0.00	– 0.06
4	0.92	0.36	– 3.71	0.95
5	1.42	1.50	1.48	1.89
6	1.41	1.82	1.89	2.00
7	1.36	1.86	1.93	1.23
8	1.89	– 0.91	– 1.15	– 1.10
9	3.29	0.07	– 0.12	– 0.16
10	3.38	1.11	1.03	0.73
11	3.77	2.16	2.05	1.56
12	1.66	2.48	2.65	2.37
13	2.35	3.11	3.36	2.89
14	3.43	2.62	2.93	2.93
15	4.39	2.81	3.29	2.94
16	4.93	3.50	4.14	3.58
17	5.60	3.53	4.58	3.77
18	5.23	4.53	5.95	4.55
19	5.86	4.36	5.91	4.47
20	6.86	5.50	7.31	4.91
21	7.03	6.33	8.55	5.71
22	8.59	8.03	11.23	6.68
23	9.03	9.12	13.36	7.62
24	11.09	9.07	13.87	7.50
25	13.20	11.06	16.99	9.02
26	17.13	12.34	18.73	10.39
27	19.65	14.21	21.49	11.99
28	19.61	15.66	23.55	13.37
29	23.66	17.41		14.60
30				16.97
31				19.03
32				19.38

process, which describes the percentage loss, is known and equal to 100%.

To identify the model providing the best fit to the battery cells degradation data in Table 1 among the ones presented in Section 2, we consider four different BTWPs: (*i*) two different models with two change-points, namely one with diffusion function $\Lambda_\sigma(t)$ in Eq. (13) and drift function $\Lambda_\mu(t)$ in Eq. (11), and one with $\Lambda_\sigma(t)$ in Eq. (12) and $\Lambda_\mu(t)$ in Eq. (11); (*ii*) one model with one change-point, with $\Lambda_\sigma(t)$ in Eq. (10) and $\Lambda_\mu(t)$ in Eq. (8); (*iii*) one model with no change-point with $\Lambda_\sigma(t)$ in Eq. (13) and $\Lambda_\mu(t) = e^{t/b} - 1$. As per all the models, $g(w)$ is modelled as in Eq. (2).

For all of them, we have computed the ML estimates $\hat{\boldsymbol{\xi}}$ of parameters, and evaluated the Akaike Information Criterion index (AIC) (Akaike, 1974):

$$AIC = 2\nu - 2\ln\left(\mathcal{L}(\hat{\boldsymbol{\xi}}, \boldsymbol{w})\right), \qquad (19)$$

where $\mathcal{L}(\hat{\boldsymbol{\xi}}, \boldsymbol{w})$ is the maximum value of the likelihood function, and ν is the number of parameters indexing the model.

The model to be selected is the one corresponding to the lowest AIC value.

The results are reported in Table 2, where K is the number of change points.

Table 2. Model comparison

K	$\Lambda_\sigma(t)$	$\Lambda_\mu(t)$	ν	$\ln\left(\mathcal{L}(\hat{\boldsymbol{\xi}}, \boldsymbol{w})\right)$	AIC
2	Eq. (13)	Eq. (11)	8	-169.92	355.84
2	Eq. (12)	Eq. (11)	8	-176.05	368.10
1	Eq. (10)	Eq. (8)	7	-171.50	357.00
0	Eq. (13)	$e^{t/b} - 1$	4	-178.41	364.82

The best model for battery cells of Table 1 according to AIC is the BTWP with two change-points, where the diffusion coefficient σ is phase-independent and the functions $\Lambda_\sigma(t)$ and $\Lambda_\mu(t)$ are modelled as in Eq. (13) and Eq. (11).

The ML estimates of the parameters of the selected model with two change-points are reported in Table 3.

Table 3. ML estimates of the parameters of the best model.

$\hat{\mu}$	4.047	$\hat{c}$	223.4
$\hat{\sigma}$	2.473	$\hat{d}$	1221 cycles
$\hat{a}$	0.5809	$\hat{t}_{c1}$	7.00 cycles
$\hat{b}$	13.43 cycles	$\hat{t}_{c2}$	18.5 cycles

Figure 2 shows the ML estimates of the mean function of the percentage loss of capacity of the battery cells together with the corresponding empirical estimates (which are computed at the inspection times only). The estimated change-point locations $\hat{t}_{c1}$ and $\hat{t}_{c2}$ are also reported. This figure gives evidence that the estimated mean of process fits very well the corresponding empirical estimates, somewhat demonstrating the suitability of the adopted degradation model.

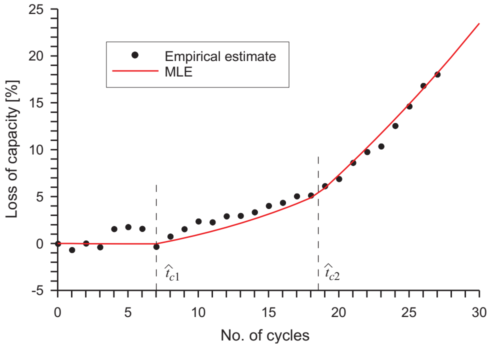

Fig. 2. Empirical and ML estimates of the mean values of the percentage loss of capacity of lithium-ion battery cells. The estimated change-point locations $\hat{t}_{c1}$ and $\hat{t}_{c2}$ are also depicted.

The same estimates are reported also in Figure 3, yet here the ML estimate of the mean function is depicted up to the cycle 75 to show how the estimated BTWP describes the long-term evolution of the mean percentage loss of capacity of the battery cells.

As mentioned in Section 3, from the ML estimates of the model parameters reported in Table 3 it is possible to easily obtain the ML estimate of any function thereof.

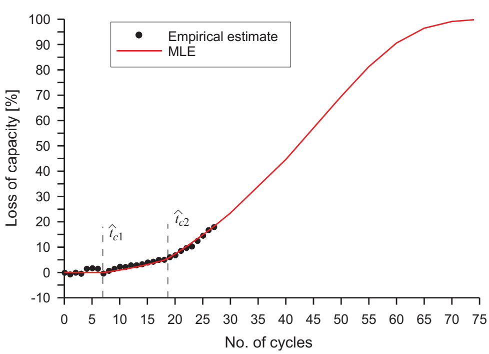

Fig. 3. Empirical and ML estimates and of the mean function of the percentage loss of capacity. Here the estimate of the mean is represented up to 75 cycles. The estimated change-point locations $\hat{t}_{c1}$ and $\hat{t}_{c2}$ are also depicted.

For example, Figure 4 shows the ML estimates of the conditional pdf of the increments of capacity loss of the four battery cells over a future interval of size $\Delta t = 10$ cycles computed at the last inspection time, conditional to the last loss reduction measurement.

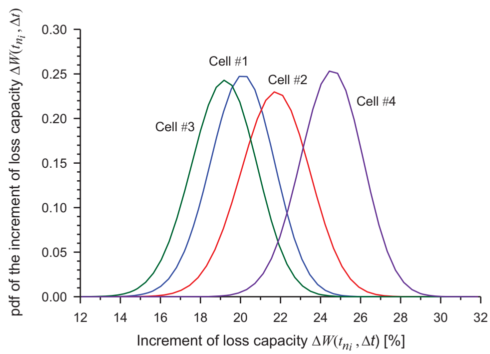

Fig. 4. ML estimates of the conditional pdf of the increment of the percentage loss of capacity of the four battery cells computed at t_{n_i}, over a future interval of size $\Delta t = 10$ cycles, given $W(t_{n_i}) = w_{i,n_i}$.

Finally, Figure 5 shows the ML estimates of the (marginal) pdf of the loss of capacity at 30, 40 and 50 cycles.

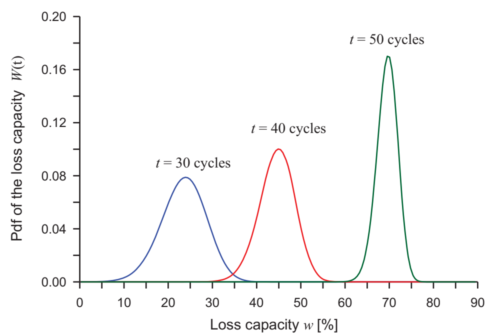

Fig. 5. ML estimates of the (marginal) pdf of the percentage loss of capacity at 30, 40 and 50 cycles.

5. Conclusions

In this paper we propose some degradation models with one or two change-points, where the evolution of the degradation over time is modelled and explained by assuming that the whole degradation phenomenon can be described by a bounded above transformed Wiener process where the functional form of the drift functions is phase specific, the drift coefficient takes one and the same value in all the phases, the diffusion function has the same functional form in all the phases, and the diffusion coefficient takes a value that could depend on the phase.

The proposed degradation models are adapted to describe bounded above non-monotonically increasing degradation phenomena that exhibits change-points.

The main properties of the model have been illustrated. The distribution of the degradation level at a given time and the conditional distribution of the future degradation increment have been formulated. The model parameters, which include the change-points location, has been estimated by using a numerical maximum likelihood estimation method.

The utility of the model and the feasibility of the proposed maximum likelihood estimation procedure are demonstrated via a numerical application that have been developed by using a real set of degradation data of lithium-ion batteries.

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