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Explanation of Ellsberg's and Machina's Four-Color Paradoxes for Judgment under Uncertainty and Comparison

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The Ellsberg's and Machina's paradoxes, involving colored balls - two, three, four, or more - continue to challenge canonical theory of decision making under uncertainty and comparison. Despite numerous models attempting to explain them, their joint explanation remains an open question, and they are often used as input filters to test models of decision making under uncertainty. The additive (with memorizing) and subtractive (with disregarding) models of cognition proposed in this paper provide the necessary means of explanation and open the prospect of unifying modeling of various biases. These models have already been applied to other cognitive biases, such as Ellsberg's two- and three-color paradoxes, as well as conjunction and disjunction fallacies. Uncertainty should be defined as probability in the context of the entire system and its set of possible states, perceived as real and imaginary images. These images depend on knowledge about them, on the randomness of their development, on the subjectivity of their assessment and are related to the structure of the human thought process. The cognitive context can be viewed and developed as possible dynamic constructions of symptoms (balls of a certain color) with varying degrees of significance. The paper presents a heuristic modeling and resolution of cognitive biases, the four-color paradoxes of Ellsberg and Machina, using the procedure for qualification and quantification of context with the performance evaluation of teamwork method. This human reliability assessment technique involves a symptom-based procedure for assessing the context probability and a step-by-step rational interpretation of cognitive and decision-making processes under conditions of uncertainty and comparison. The core idea behind overcoming cognitive biases is to exploit quantum principles and dual role of a symptom as a wave and a chunk of information in conscious processing, with delays, ambiguity, and violations.

Keywords: Bias, Context, Symptom, Judgment, Uncertainty, Risk, Ambiguity, Paradoxes of Ellsberg and Machina.

1. Introduction

Historically, research on decision making under uncertainty has focused on various distortions, biases, and exaggerations of the influence of risk, utility, or profit, depending on the application domain. Bernoulli (1738) observed that most people are risk-averse and used his psychological insights to propose a fundamentally novel approach to evaluating gambling in terms of the utility of wealth. Knight (1921) attempted to address the problem of risk aversion for gain by distinguishing between risk (measurable) and ambiguity (immeasurable) uncertainty. Von Neumann and Morgenstern (1947) represented risk using a known objective probability (OP), and Savage (1954) attempted to define personally estimated subjective probability (SP). However, to properly define SPs success or failure, it should not be assumed that they are related solely to indicators of utility and risk, as they do not fully describe the construction of cognitive processes. Modeling decision-making processes is related to changes in information entropy (Shannon, 1948), which postulates that the brain is capable not only of selectively suppressing task-irrelevant information, but also of simultaneously or sequentially eliminating it from use. The decision-maker must clearly distinguish "task-irrelevant" from "task-relevant" stimuli and compare "clear and vague prospects jointly" (Fox and Tversky, 1995). Therefore, to avoid cognitive biases and explain uncertainty aversion, they must be identified, interpreted, qualified, and quantified in the context of the entire socio-technical system (STS).

Daniel Ellsberg (1961) also demonstrated in a series of thought experiments that knowers and decision makers typically prefer clear to vague information. Ellsberg's paper also cited another example of Ellsberg's four-color paradox (Machina, 2009), suggested to him by Kenneth Arrow. This example was subsequently used to expand and refine Machina's concept of uncertainty aversion (Machina, 2014). The Machina paradox appears because none of the reviewed by him models can represent this dual behavior. The Ellsberg's and Machina's paradoxes, involving colored balls - two, three, four, or more - continue to challenge canonical theory of decision making under uncertainty and comparison. Despite numerous models attempting to explain them, their joint explanation remains an open question, and they are often used as input filters to test models of decision making under uncertainty (Sharpe, 2023). Consequently, the construction of a unified framework explaining risk and ambiguity is still open problem in decision making (Buchak, 2025). This uncertainty aversion is crucial for risk assessment, management, and communication, and especially for the widely used expert judgment in human reliability assessment (HRA).

The current paper presents heuristic modeling and resolution of cognitive biases or misjudgment in ambiguous and comparative contexts, using the procedure for qualification and quantification of context with the *performance evaluation of teamwork* (PET) method. Based on the PET definition of the STS context with symptoms as *object versus subject in a situation*, several important cognitive biases have been investigated and resolved: conjunction (Petkov, 2023) and disjunction (Petkov, 2024) fallacies, as well as Ellsberg's two-color (Petkov et al, 2025) and three-color paradoxes (Petkov, 2025). In this paper they are modeled and resolved the four-color Ellsberg and Machina paradoxes for judgment in ambiguous and comparative contexts.

2. Stepwise modeling of cognition

As shown in previous papers, the process of cognition can be modeled by discrete successive steps (2, 4 or more) of approximation of the subjectively recognized number of symptoms (balls of assorted colors) to their objective number. The additive (with memorizing) and subtractive (with disregarding) models of cognition proposed before and here provide the necessary means of explanation and open the prospect of unifying modeling of various biases.

In the four-color problems of Ellsberg and Machina, the following two models of stepwise cognition are used, shown in Table 1:

1. *Cumulative cognition with accumulation* (CC_a) in two steps - without forgetting the recognized symptoms, objectively and subjectively, preserving their number; The scheme of CC_a is presented in Table 1 in grey, where $N + M$ is the total number of balls in the urn with different colors $i = 1 \ldots 4$ (1-black, 2 - red, 3 - green and 4 - yellow).
2. *Subtractive cognition with disregarding* (SC_d) in 4 steps, j=1…4, with neglecting the recognized symptoms, objectively and subjectively, by reducing their numbers.

The SC_d scheme is presented in Table 1 in grey (1st and 2nd step) and white (3rd and 4th steps), where 'V' means symptom (ball) with violation or distortion because of payoff: *O* and *S* mean *objective* or *subjective* number of symptoms, respectively. In the four-color Ellsberg and Machina 'reflection' problems - M=N, and in the four-color Machina '50:51' problem M=N+1.

Table 1. Stepwise models of cognition:
CC_a with memorizing and SC_d with disregarding.

Color 1	Step 1	Step 2	Step 3	Step 4
V1	Z_{11}	Z_{12}	Z_{13}	Z_{14}
O1	n	n	n-1	n-1
S1	0	1	1	0
Color 2	**Step 1**	**Step 2**	**Step 3**	**Step 4**
V2	Z_{21}	Z_{22}	Z_{23}	Z_{24}
O2	N-n	N-n	N-n-1	N-n-1
S2	0	1	1	0
Color 3	**Step 1**	**Step 2**	**Step 3**	**Step 4**
V3	Z_{31}	Z_{32}	Z_{33}	Z_{34}
O3	M	m	m-1	m-1
S3	0	1	1	0
Color 4	**Step 1**	**Step 2**	**Step 3**	**Step 4**
V4	Z_{41}	Z_{42}	Z_{43}	Z_{44}
O4	M-m	M-m	M-m-1	M-m-1
S4	0	1	1	0

3. Four-color Ellsberg paradox

3.1. *Four-color Ellsberg problem description*

The example, known as the *four-color Ellsberg paradox*, is presented in (Ellsberg, 1961, p.654, note 4). It involves an urn containing balls of four colors (e.g., black, red, green, and yellow). The urn contains *2*N* balls (*200 balls in Ellsberg's experiment),* where *n* black, *N-n* red, *N/2* green and *N/2* yellow balls, where *n* ϵ *[0, N]*. It means that the vague substance of the urn contains *n* black and *(N-n)* red balls in an unknown proportion. One ball will be drawn from the urn. A person is asked to bet on one of the acts *b1*, *b2*, *b3* and *b4* defined in Table 2.

Table 2. Balls' number, colors, and payoff matrix of the Ellsberg "four-color" problem.

bet	*'vague'*		*'clear'*	
color	*black*	*red*	*green*	*yellow*
number of balls	*n*	*N-n*	*N/2*	*N/2*
b1, game 1	2*Z	2*Z	0	0
b2, game 1	2*Z	0	2*Z	0
b3, game 2	0	2*Z	0	2*Z
b4, game 2	0	0	2*Z	2*Z

Suppose players are offered two games to be played as follows:

1) *Comparative objective-subjective game.* The player must guess the color and bet on black/red (*b1*) vs black/green (*b2*), then select a ball from the urn.
2) *Comparative subjective-objective game.* The player must guess one of two colors – red/yellow (*b3*) vs green/yellow (*b4*), then select a ball from the urn.

If the color that player draws is the same as the she/he predicted, then player will win *a=$2*Z (Z>0)*; otherwise, she/he will win nothing, *b=$0 (in Ellsberg's experiment a=$100, b=$0)*. To simplify the task of the four-color problem of Ellsberg and Machina, the unit of payoff Z, is chosen to be the least common multiple of Machina's experiments (*Z=$4000*), which is equivalent to one symptom as one ball. In the four-color problem of Ellsberg, *Z=$50* (80 times less).

The choice of Z is subjective for each agent. Varying Z can also change the direction of the ambiguity aversion, as in the examples of Ellsberg and Machina (by modulating the waves of symptoms). If ambiguity is important to the decision maker, the magnitude of the difference between the subjective Z of two agents can determine whether it will compensate for the differences in the *subjectively estimated probability measures* from the different numbers of balls in the experiments. This allows us to use a dimensionless quantity by adding an additional symptom-wave. It is a third column (V_i - violation column, *i=1…4*) of the number of balls of a certain color.

In the original PET method, for HRA, such an approach is used to model symptom violations (distortions), which worsens the context and error probabilities. But the peculiarity of modeling the payoff matrix is that this wave should have the opposite effect, increasing the probability of making a given judgment. Therefore, the violation should be placed on the color of ball for that pay less after drawing, based on the symmetry of the payoff matrix (Tables 2, 8 and 11, where Z_{ij}, *j=1…4*).

3.2. *Principal results*

The Ellsberg's four-color problem, as well as Ellsberg's two-color (Petkov et al, 2025) and three-color (Petkov, 2025) problems, show that individuals' preferences deviate from the classical *Subjective Expected Utility* (SEU) model by exhibiting a systematic preference for objective over subjective bets and typical rankings *bet 1 > bet 2* ($OP_{b1} > SP_{b2}$) and *bet 3 < bet 4* ($SP_{b3} < OP_{b4}$) respectively reveal the same type of a phenomenon known as *ambiguity aversion*. Ellsberg observed that such examples can be viewed as providing systematic violations of Savage's *Sure-Thing Principle,* postulate P2 (Savage, 1954, p.23). Such examples have stimulated the development of alternatives to SEU model, most notably the *rank-dependent expected utility* or *Choquet expected utility* model, which describes preferences for subjective bets.

3.3. *PET results*

The results of PET and comparison of bets for the ratio of the sums of *subjectively estimated probability measures* of a successful decision to select a ball from the "clear" and "vague" parts the urn for the comparative *objective-subjective* (*b1* versus *b2*) and *subjective-objective* (*b3* vs *b4*) games are presented in Table 3, Table 4 and Figure 1 for '(2+2),1': N=2 and 2*Z=1. Similar PET comparisons are obtained in the cases for '(4+4),2' and '(8+8),2' in Table 5 and Figure 2, Table 6 and Figure 3, and for original four-color Ellsberg's paradox '(100+100),2' in Table 7 and Figure 4. As the number of balls increases, the inequalities remain valid only for the SC_d model, in which, due to the presence of subjective recognition, it ends at the 4th step of cognition.

Table 3. The PET results for the ‘(2+2),1’ Ellsberg’s four-color paradox.

Model	b1	b2	b3	b4
Step	SC_d	SC_d	SC_d	SC_d
Step 1 CC_a=SC_d	0,1019	0,0972	0,0972	0,1007
	0,1019	0,0972	0,0972	0,1007
	0,1019	0,0972	0,0972	0,1007
	0,1019	0,0972	0,0972	0,1007
Step 2 CC_a=SC_d	0,1667	0,1528	0,1667	0,1597
	0,1667	0,1667	0,1528	0,1701
	0,2037	0,1944	0,1944	0,2014
	0,2037	0,1944	0,1944	0,2014
Step 3	0,2593	0,2361	0,2222	0,2083
	0,2593	0,2222	0,2361	0,2396
	0,3056	0,3889	0,2917	0,4028
	0,3056	0,2917	0,3889	0,4028
Step 4	0,1481	0,1389	0,1250	0,1250
	0,1481	0,1250	0,1389	0,1354
	0,1528	0,1944	0,1458	0,2014
	0,1528	0,1458	0,1944	0,2014

Table 4. The PET comparison for ‘(2+2),1’ case of Ellsberg’s four-color paradox.

bets \ steps	step 1	step 2	step 3	step 4
bet 1 objective	0,2037	0,3334	0,5185	0,2963
bet 2 subjective	0,1944	0,3194	0,4583	0,3333
bet 3 subjective	0,1944	0,3472	0,6250	0,3333
bet 4 objective	0,2014	0,4028	0,8056	0,4028
model	CC_a=SC_d	CC_a=SC_d	SC_d	SC_d

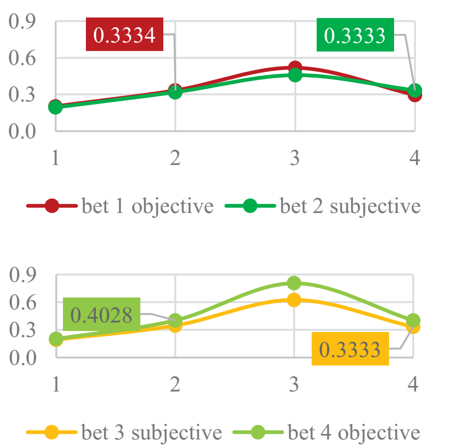


Figure 1. The PET stepwise cognition for b1vs.b2 & b3vs.b4 for ‘(2+2),1’ of Ellsberg’s four-color paradox.

Of course, there is always the possibility that it ends at a previous step and that the four-color Ellsberg paradox is not observed. The value of Z is highly individual and here the dependence on its absolute value is not studied, but only the ratio between the payoffs for the different ball colors.

Table 5. The PET comparison for ‘(4+4),2’ case of Ellsberg’s four-color paradox.

bets \ steps	step 1	step 2	step 3	step 4
bet 1 objective	0,061	0,106	0,149	0,083
bet 2 subjective	0,056	0,099	0,13	0,072
bet 3 subjective	0,056	0,103	0,152	0,081
bet 4 objective	0,049	0,097	0,146	0,073
model	CC_a=SC_d	CC_a=SC_d	SC_d	SC_d

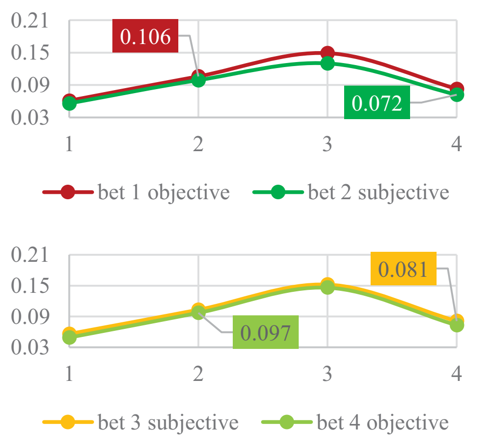


Figure 2. The PET stepwise cognition for b1vs.b2 & b3vs.b4 for ‘(4+4),2’ of Ellsberg’s four-color paradox.

Table 6. The PET comparison for ‘(8+8),2’ case of Ellsberg’s four-color paradox.

bets \ steps	step 1	step 2	step 3	step 4
bet 1 objective	0,023	0,042	0,054	0,029
bet 2 subjective	0,021	0,038	0,047	0,025
bet 3 subjective	0,022	0,039	0,05	0,026
bet 4 objective	0,018	0,035	0,043	0,022
model	CC_a=SC_d	CC_a=SC_d	SC_d	SC_d

Using the PET procedure for calculating *context probability* (CP) or contexture, we can calculate the sums of *subjectively estimated probability measures* for a successful judgment. These are determined by a conditionally alternative distribution of balls, actions, and bets, the lower bound of which is SP=(1-CP).

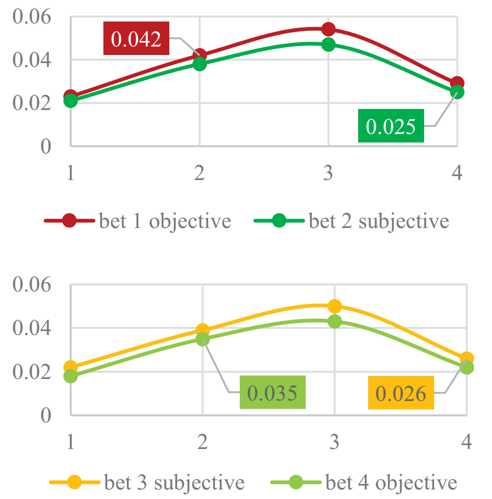


Figure 3. The PET stepwise cognition for b1 vs. b2 and b3 vs. b4 for '(8+8),2' of Ellsberg's four-color paradox.

Table 7. The PET comparison for '(100+100),2' case of Ellsberg's four-color paradox.

bets \ steps	step 1	step 2	step 3	step 4
bet 1 objective	7,98E-05	1,53E-04	1,71E-04	8,88E-05
bet 2 subjective	6,85E-05	1,32E-04	1,44E-04	7,43E-05
bet 3 subjective	7,05E-05	1,37E-04	1,43E-04	7,62E-05
bet 4 objective	5,89E-05	1,18E-04	1,20E-04	6,00E-05
model	CC_a=SC_d	CC_a=SC_d	SC_d	SC_d

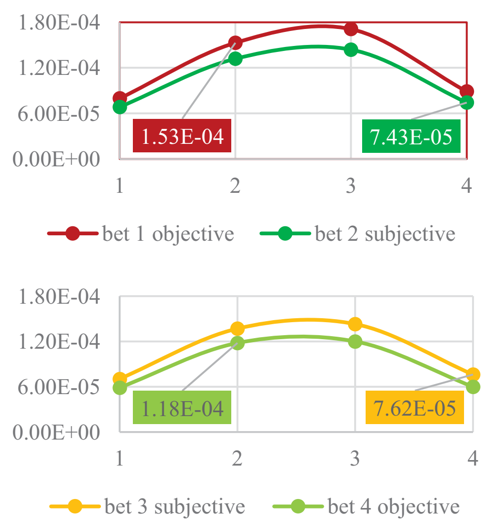


Figure 4. The PET stepwise cognition for b1vs.b2 & b3vs.b4 for '(100+100),2' Ellsberg's 4-color paradox.

The preference results and SP values obtained using the PET method confirm Ellsberg's conclusion that the 'clear' over 'vague' bets and typical rankings *bet 1* > *bet 2* and *bet 3* < *bet 4* as follows from the Tables 4, 5, 6, and 7.

4. Four-color Machina paradoxes

4.1. *Machina's 'N:(N+1)' example description*

The Machina paradox (2009) experiment considered an urn containing *2*N+1* (*101 in Machina's experiment*) balls, each numbered from 1 to 4 (e.g., black, red, green, and yellow). The number of balls with the number 1 (black) plus the number of balls with the number 2 (red) equals *N* (*50 in Machina's experiment*), and the number of balls with the number 3 (green) plus the number of balls with the number 4 (yellow) equals *N+1* (*51 in Machina's experiment*), where $n \in [0, N]$ and $m \in [0, N+1]$. A person is asked to bet on one of the acts *b1*, *b2*, *b3* and *b4* defined in Table 8. The event E_j indicates that a ball with a number *j* has been drawn from the urn, the act b_1 has been defined as contingent payoff in each event, so that in E_1, b_1 pays *2*Z = $8,000*, in E_2, b_2 pays *2*Z = $8,000*, and so on. Equally are defined bets: b_3 and b_4. Then, free of charge a person is asked to bet on b_1 or to bet on b_2, if he or she is sufficiently uncertainty averse then will prefer b_1 instead b_2, because b_1 has no ambiguity in its payoffs although b_2 presents a slight Bayesian advantage due to the *N+1* (*51*) balls may yield *2*Z = $8,000*. The person is also asked to bet on b_3 or b_4. In this case, both acts present ambiguity in their payoffs, there is not an informational advantage between them. Thus, a decision–maker who values unambiguous information would be indifferent between b_3 and b_4. On the other hand, b_4 benefits from the *N+1* (*51*) balls, hence in this case the Bayesian advantage implies that $b_4 > b_3$.

Table 8. Balls' number, colors, and payoff matrix of the Machina's four-color 'N:(N+1)' problem.

act (bet)	*'vague' (N)*		*'vague' (N+1)*	
color	E_1 *black*	E_2 *red*	E_3 *green*	E_4 *yellow*
number of balls	n	N-n	m	N+1-m
I - b1	2*Z	2*Z	Z	Z
II - b2	2*Z	Z	2*Z	Z
III - b3	3*Z	2*Z	Z	0
IV - b4	3*Z	Z	2*Z	0

4.2. *Principal results for 'N:(N+1)' example*

In the example 'N:(N +1)', b1 and b2, as well as b3 and b4, differ only in whether they offer the higher reward 2*Z in the event E2 or E3. If the decision maker is sufficiently wary of ambiguity, he/she will prefer b1 to b2, as Machina claims. In fact, b1 is clearly unambiguous, while b2 is ambiguous but benefits from a slight advantage due to the (N+1)-th ball, which can lead to 2*Z. There is therefore a trade-off between the advantage offered by b2 and the lack of ambiguity offered by b1. Such a trade-off is less clear in the choice between b3 and b4. Like b2, b4 benefits from the (N+1)-th ball, but b3 does not offer a specific informational advantage. In this example, there are two conflicting choices. However, a decision maker who values unambiguous information may prefer b1 to b2 and may be indifferent between b3 and b4. The information advantage of b1 may more than compensate for its Bayesian disadvantage over b2, while b3 does not benefit from a clear information advantage that could compensate for its Bayesian disadvantage over b4. This would lead to b1 > b2 and b3 < b4. If the 'N:(N +1)' ball distribution does not provide the "right advantage" or if it provides "too much advantage", this can be corrected, either by changing (decreasing / increasing) N or if necessary to reduce/increase the advantage 2*Z.

4.3. *PET results for 'N:(N+1)' example*

The results of PET and the comparison of the ratios of the sums of *subjectively estimated probability measures* for successfully judging the choice of a ball from the N and (N+1) parts of the urn, confirming Machina's conclusion, for both the models and the actions/bets 1÷4 are presented in Table 9 (N=2), Table 10 and Figure 5.

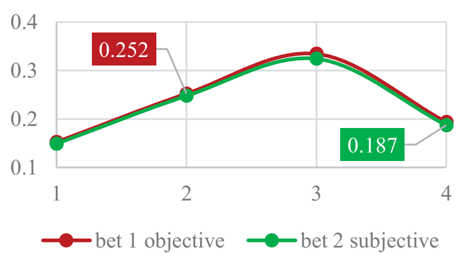
When solving the Machina's four-color 'N:(N+1)' problem in this paper, since N=2 is an extreme case with minimal N. The correct ratio, even with this increase in N to 50, will be observed also guaranteed by using the SC_d model as well, since the choice of the payoff Z is conditional and subjective. In fact, in Machina's '50:51' problem with four colors it is Z=$4000, which is 80 times more than the assumed Z=$100, to be like the Ellsberg's problem with four colors (Baillon et al, 2011).

Table 9. Machina's '2:(2+1)' PET results.

Model	b1	b2	b3	b4
Step	SC_d	SC_d	SC_d	SC_d
Step 1 CC_a=SC_d	0,0762	0,0744	0,0997	0,0990
	0,0762	0,0744	0,0997	0,0991
	0,0762	0,0744	0,0997	0,0991
	0,0762	0,0744	0,0997	0,0991
Step 2 CC_a=SC_d	0,1261	0,1210	0,1567	0,1526
	0,1261	0,1256	0,1710	0,1728
	0,1324	0,1267	0,1717	0,1668
	0,1324	0,1311	0,1751	0,1762
Step 3	0,1671	0,1597	0,2422	0,2346
	0,1671	0,1551	0,2279	0,2144
	0,1615	0,1644	0,2093	0,2165
	0,1615	0,1600	0,2058	0,2072
Step 4	0,0968	0,0938	0,1459	0,1815
	0,0968	0,0892	0,1282	0,1198
	0,0908	0,0933	0,1185	0,1238
	0,0908	0,0889	0,1150	0,1145

Table 10. The PET comparison for the '2:(2+1)' case of the Machina's four-color paradox.

bets \ steps	step 1	step 2	step 3	step 4
bet 1 objective	0,152	0,252	0,334	0,194
bet 2 subjective	0,149	0,248	0,324	0,187
bet 3 subjective	0,199	0,328	0,470	0,274
bet 4 objective	0,198	0,319	0,451	0,305
model	CC_a=SC_d	CC_a=SC_d	SC_d	SC_d



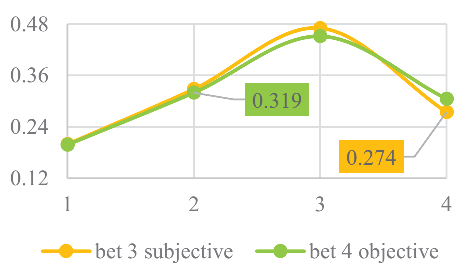


Figure 5. The PET stepwise cognition for b1 vs. b2 and b3 vs. b4 for '2:(2+1)' of Ellsberg's four-color paradox.

4.4. *Machina's 'reflection' example description*

As in the '50:51' example, and in the 'reflection' example, shown in Table 11, the ambiguity-averter's choice may depend on her/his trade-off rate between this objective and subjective uncertainty, as well as on her/his attitude toward the different payoff levels. If his personal trade-off rate exactly matched that of the problem, he might be indifferent.

Table 11. Balls' number, colors, and payoff matrix of the Machina four-color 'reflection' problem.

act (bet)	*'vague' (N)*		*'vague'(N)*	
color	*E_1 black*	*E_2 red*	*E_3 green*	*E_4 yellow*
number of balls	N	N-n	n	N-n
I – b5	Z	2*Z	Z	0
II – b6	Z	Z	2*Z	0
III – b7	0	2*Z	Z	Z
IV – b8	0	Z	2*Z	Z

4.5. *Principal results for 'reflection' example*

The experimental Machina's 'reflection' example conducted by L'Haridon and Placido (2010) showed a typical preference pattern of b5 < b6 and b7 > b8 (for 46% of participants), although 28% of participants showed b5 > b6 and b7 < b8. Therefore, it can be expected that the proposed PET models for ambiguity aversion should explain the following results for this Machina's 'reflection' example: b5 < b6 and b7 > b8. This is verified for the two PET models (CC_a and SC_d) of cognition considered in this paper.

4.6. *PET results for 'reflection' example*

The PET results and the comparison of the ratios of the sums of *subjectively estimated probability measures* of a successful judgment for choosing a ball from the urn for both models and I÷IV acts are presented in Table 12 (N=2) and Table 13 and Figure 6.

The preference results and SP values obtained using the PET method confirm Machina, L'Haridon and Placido (2010) conclude that the typical rankings bet 5 < bet 6 and bet 7 > bet 8 as follows from Table 13. The uncertainty avoidance axiom is the driving force behind most of the models and results in this paper.

Table 12. Machina's 'reflection' PET results.

Model	**b1**	**b2**	**b3**	**b4**
Step	**SC_d**	**SC_d**	**SC_d**	**SC_d**
Step 1 CC_a=SC_d	0,1556	0,1575	0,1575	0,1556
	0,1444	0,1448	0,1481	0,1444
	0,1444	0,1448	0,1481	0,1444
	0,141	0,1402	0,1446	0,1398
Step 2 CC_a=SC_d	0,2667	0,2605	0,2748	0,2625
	0,2444	0,2605	0,2427	0,2528
	0,2528	0,2427	0,2605	0,2444
	0,2556	0,2656	0,2535	0,2574
Step 3	0,3556	0,3453	0,341	0,3241
	0,3778	0,3453	0,3731	0,3338
	0,3338	0,3731	0,3453	0,3778
	0,3148	0,3287	0,3348	0,3417
Step 4	0,2	0,1999	0,1906	0,1863
	0,2222	0,1999	0,2227	0,1961
	0,1961	0,2227	0,1999	0,2222
	0,1817	0,1844	0,1947	0,1931

Table 13. The PET comparison for the 'reflection' case of the Machina's four-color paradox.

bets \ steps	**step 1**	**step 2**	**step 3**	**step 4**
bet 5 v-vN	0.3	0.511	0.733	0.422
bet 6 v-cZ	0.302	**0.521**	**0.691**	0.4
bet 7 v-vN	0.293	0.496	0.708	0.417
bet 8 c-vZ	0.284	0.502	0.719	0.415
model	**CC_a=SC_d**	**CC_a=SC_d**	**SC_d**	**SC_d**

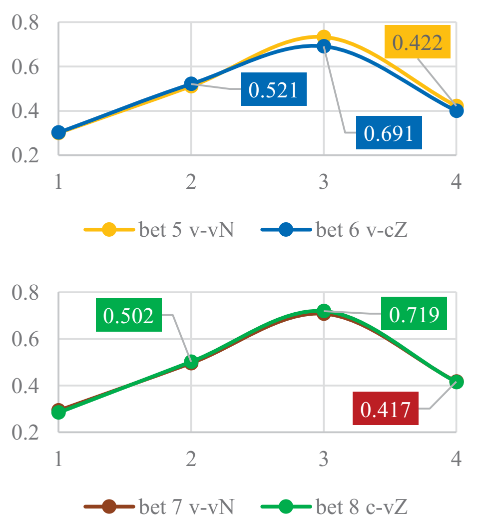


Figure 6. The PET stepwise cognition for b5vs.b6 & b7vs.b8 for Ellsberg's four-color 'reflection' paradox.

5. Conclusions

The modeling, explanation, and resolution of the four-color paradoxes of Ellsberg and Machina, as well as the two-color and three-color paradoxes of Ellsberg, allows us to present a heuristic and rational PET methodology for interpreting many experimental psychological studies of problems, fallacies, and paradoxes involving biased thinking and judgment.

The PET method allows obtaining the subjective distribution of probability amplitudes associated with the specific cognition and decision-making process for each alternative (option, act and bet).

Avoiding cognitively biased errors in judgment primarily involves limiting the effects of delayed, distorted or violated symptom recognition by carefully qualifying, grouping, and quantifying symptoms and the stepwise wavy process of cognition.

This explicit qualification and quantification of the judgment context with a different number of stimuli (symptoms as balls) allows the implementation of the PET method in a system for collecting data on human errors based on symptom-oriented emergency procedures, which are most widely used in nuclear power engineering. A pilot project to create such a system is being built at the Kozloduy NPP in Bulgaria based on data from the regular training of operator crews on the full-scale VVER-1000 simulator.

The inability to complete the process of cognition and judgment in ambiguous and comparative contexts is due to the alternating additive and subtractive construction of the processes of human cognition and decision-making in conditions of delayed, distorted or stochastic possibilities for encoding information in working memory.

This, in turn, limits subsequent access to this information in long-term memory, where the *"individual judgment processors"* are stored and used through a unified probabilistic-deterministic logic of thinking.

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