Proc. of the European Safety and Reliability Conference (ESREL 2026)
Edited by José C. Matos, Paulo B. Lourenço, Michael Beer, Daniel Oliveira, Edoardo Patelli, Hélder S. Sousa, Rui A. Silva and Monica Santamaria
 Published by Research Publishing, Singapore.
doi: 10.3850/ESREL2026061419_esrel26-p26408-cd

Minimizing CO_2 Emissions Through Predictive Maintenance: A Stochastic Modeling Approach

Elham Mosayebi Omshi
LIST3N-MSAD, Université de Technologie de Troyes, Troyes, France. E-mail: elham.mosayebi@utt.fr

Antoine Grall
LIST3N-MSAD, Université de Technologie de Troyes, Troyes, France. E-mail: antoine.grall@utt.fr

Industrial CO_2 emissions are influenced by both deterministic factors, such as energy consumption and operational processes, and stochastic variables, including equipment degradation and unforeseen system failures. In this study, we model CO_2 emissions as a function of system degradation, where the cumulative emissions up to time t are represented by a stochastic integral of this function. This approach enables the prediction of emission trajectories under uncertain system conditions, capturing the inherent randomness in operational dynamics. The primary objective of this paper is to determine an optimal maintenance policy that minimizes both maintenance costs and cumulative CO_2 emissions. To achieve this, we develop a mathematical framework that integrates stochastic degradation modeling with maintenance optimization techniques. The proposed model is validated through a simulation study, demonstrating its effectiveness in balancing economic and environmental objectives. The results offer valuable insights into sustainable industrial operations by providing strategies for proactive maintenance that mitigate emissions while ensuring cost efficiency.

Keywords: Predictive maintenance, deteriorating systems, carbon emissions.

1. Introduction

Maintenance actions, ranging from minor repairs to full replacements, are critical for keeping systems operational. Anticipating failures and performing maintenance proactively can reduce downtime and operational costs. Maintenance modeling provides a framework for planning monitoring, and maintenance activities throughout the equipment's life cycle, guiding decisions on what actions to take and when.

With advances in monitoring technologies, maintenance policies are increasingly based on the observed condition of the system. In this context, stochastic models provide an appropriate and rigorous framework for degradation modeling, as system deterioration is inherently driven by random mechanisms and uncertain operating conditions. Stochastic degradation models, such as gamma, and inverse Gaussian processes, explicitly account for this uncertainty by modeling irreversible degradation as a random evolution over time. This approach enables realistic representation of degradation paths, supports probabilistic prediction of failure times, and facilitates the analytical derivation of reliability measures and maintenance performance indicators. See Liu et al. (2024), Bautista et al. (2022), Xu et al. (2025), Fang et al. (2022) for some recent studies in this context.

In classical maintenance research, the primary objective is to identify an optimal maintenance policy aimed at reducing total maintenance costs. However, with the increase in standards to be met in the safety and environmental areas, cost minimization that might entail increasing environmental risk or human damage is not acceptable. Therefore, in addition to the more specific objectives within an organization, maintenance managers cannot neglect environmental and safety goals. It is important to note that these aspects conflict, De Almeida et al. (2015). Single-objective optimization approaches fail to capture these inherent trade-offs, whereas multi-objective optimization provides a comprehensive framework for simultaneously addressing economic, technical, and environmental criteria. Peng et al. (2022) presented a framework to optimize the maintenance strategy of a deteriorating bridge considering sustainability criteria, including maximizing safety performance and minimizing economic, social, and environmental impacts. Choi (2019) compared road maintenance strategies using life-cycle cost and environmental assessments, identifying the most economical and carbon-efficient option when CO_2 emissions are included.

By generating Pareto-optimal solutions, this approach enables decision-makers to evaluate efficient trade-offs among reliability, cost, safety, and environmental sustainability, thereby allowing them to select the most appropriate solution based on practical priorities. Levitin (2006) presented computational intelligence and evolutionary techniques in reliability engineering that naturally address multi-objective optimization problems, where Pareto frontiers are used to identify optimal trade-offs among reliability, cost, and system performance.Certa et al. (2011) determined the Pareto frontier in multi-objective maintenance optimization to analyze trade-offs between maintenance cost and system reliability. Wang and Pham (2011) proposed a multi-objective maintenance optimization model for a single-unit system under competing risks, using imperfect preventive maintenance and NSGA-II to balance availability and cost through Pareto-optimal strategies.

In this study, we focus on a multi-objective maintenance policy that jointly minimizes maintenance costs and CO_2 emissions. CO_2 emissions have a significant environmental impact, as they contribute to global warming and climate change. Reducing these emissions is therefore essential to mitigate environmental damage, protect natural ecosystems, and promote a more sustainable future. Degraded systems typically require more energy or resources to operate, which leads to increased carbon emissions. Accordingly, we assume that CO_2 emissions during a maintenance cycle depend on the system's condition. The accumulated carbon emissions are modeled using stochastic integrals, whose properties are exploited to derive expressions for the long-term cost rate and cumulative emissions. The performance of the proposed maintenance policy is then investigated through numerical analysis.

The rest of the paper is structured as follows. Section 2 provides general assumptions on the system degradation modeling. Section 3 deals with maintenance policy description and analytical derivations for cost assessment. Section 4 presents the multi-objective formulation and details the calculations associated with each objective function. Section 5 is devoted to the numerical study of the proposed policy. Finally, Section 6 concludes the paper with directions for further researches

2. Degradation Model

Consider a single-component system subject to degradation. Let X_t denote the system's degradation level at time t. In the absence of maintenance actions, X_t evolves as a strictly increasing stochastic process with $X_0 = 0$. The system fails when the degradation exceeds a critical threshold L. Failures are not self-announcing, so the system remains failed until the next inspection, incurring additional downtime costs.

We assume that X_t follows a gamma process with independent, nonnegative increments such that, for each $t > s \geq 0$, the increment $Y = X_t - X_s$ follows a gamma distribution $G(\alpha(t-s), \lambda)$ with the following probability density function:

$$f(y) = \frac{\lambda^{\alpha(t-s)}}{\Gamma(\alpha(t-s))} y^{\alpha(t-s)-1} e^{-\lambda y}, \quad y > 0,$$

where α and λ are known positive parameters, and $\Gamma(\cdot)$ denotes the gamma function defined as follows:

$$\Gamma(z) = \int_0^{\infty} u^{z-1} e^{-u} du.$$

3. Maintenance Policy

We consider a periodic inspection policy with inspections conducted every T time units. At each inspection, an appropriate maintenance action is selected. To mitigate failure risk, a preventive threshold $M < L$ is defined. The possible outcomes at inspection are as follows:

- If the degradation level exceeds L, the system has failed and is correctively replaced.

- If the system has not failed but its degradation exceeds M, a preventive replacement is carried out.
- If the degradation level is below M, the system remains operational, and no action is taken.

All maintenance actions are assumed to take negligible time. Both preventive and corrective replacements are perfect, restoring the system to an "as-good-as-new" state.

4. Optimization

Here, we are going to find the optimal maintenance actions given a multiobjective optimization framework to minimize both maintenance costs and cumulative CO_2 emissions.

In other words:

$$\begin{aligned} &\text{minimize} \quad (EC(\theta), EE(\theta)) \\ &\text{subject to}: \theta \in \Theta, \end{aligned} \tag{1}$$

where $\theta = (M, T)$, and Θ is the feasible set and equal to $[0, L) \times [0, +\infty)$. The two objective functions, $EC(\cdot)$ and $EE(\cdot)$, represent the long-run average cost per unit time and the long-run average carbon emissions per unit time, respectively. Their formulations are provided in the following subsections.

The solutions to a multi-objective optimization problem can generally be classified into two main approaches: the scalarization and the Pareto-based method (De Weck (2004)). These two approaches differ in how they handle multiple conflicting objectives. The scalarization method combines multiple performance indicators into a single scalar objective function. This is usually done by incorporating the objectives into a weighted or aggregated fitness function, transforming the multi-objective problem into a single-objective one. In contrast, the Pareto-based method treats the objectives separately and seeks a set of non-dominated solutions that represent trade-offs among the objectives. The outcome is typically presented as a Pareto optimal front, which illustrates the range of compromise solutions available to the decision-maker. The main advantage of the Pareto-based method over scalarization is that it preserves the true trade-off structure of the problem by generating a set of non-dominated solutions without requiring predefined weights or reducing multiple objectives to a single aggregated function. Here, we are going to focus of the Pareto-based method.

In a multi-objective optimization problem, the Pareto front represents the set of non-dominated solutions in objective space. A solution is non-dominated if no other feasible solution improves at least one objective without worsening another. In the given problem (1), the Pareto front consists of all solutions θ^* such that there is no other solution θ for which $EC(\theta) \leq EC(\theta^*)$ and $EE(\theta) \leq EE(\theta^*)$, with strict inequality for at least one objective. The optimization with two objective functions and the non-dominated solution can be described in a two-dimensional surface.

In the Pareto method, several terms in the Pareto optimal solution need to be noted. The Utopia point is defined as $(EC^{\min}, EE^{\min})$ where each component is the best achievable value of that objective individually. Although this point is typically unattainable simultaneously, it serves as a reference for evaluating compromise solutions. A common decision rule is to select the Pareto solution closest to the ideal point, often using a distance metric such as the Euclidean distance (after normalization if objectives have different scales). This closest point represents a balanced trade-off among objectives, minimizing the overall deviation from the ideal performance.

In order to find the closest point, for each point with coordinates $(EC(\theta^*), EE(\theta^*))$ on the Pareto optimal front, the normalized Euclidean distance is defined as follows:

$$\sqrt{\left(\frac{EC(\theta^*) - EC^{\min}}{EC^{\max} - EC^{\min}}\right)^2 + \left(\frac{EE(\theta^*) - EE^{\min}}{EE^{\max} - EE^{\min}}\right)^2}.$$

The coordinates of the closest point are obtained by minimizing this distance.

The next subsections provide a more detailed development of the two objective functions.

4.1. *Maintenance Cost*

Maintenance activities involve associated costs. Inspections are scheduled at times $T_k = kT$, each incurring a cost c_i. During these inspections, a preventive maintenance action can be carried out at c_p. In the event of system failure, a corrective replacement is required, which costs c_c. Then, the cumulative cost function on $[0, t]$ is defined as:

$$C(t) = c_i N_i(t) + c_p N_p(t) + c_c N_c(t) + c_d D(t),$$

where $N_i(t)$, $N_p(t)$, and $N_c(t)$ represent the number of inspections, preventive replacements, and corrective replacements, respectively. In addition, $D(t)$ denotes the total downtime experienced by the system up to time t.

The cost function depends on the threshold value M and the time interval between maintenance actions T. To identify the optimal values of the decision variables M and T, the long-run average cost per unit time, denoted by EC, is used as the objective function to be minimized.

$$EC(\theta) = \lim_{t\to\infty} \frac{\mathbb{E}\left(C(t)\right)}{t},$$

From classical renewal theory, it is well known that for any degradation process with the regenerative property, the following equation holds.

$$EC(\theta) = \frac{\mathbb{E}(C(S_1))}{\mathbb{E}(S_1)},$$

where S_1 is the length of one renewal cycle, i.e., the time between two successive replacements. The expectations can be calculated through a Monte Carlo simulation.

4.2. *Carbon Emissions*

As systems begin to degrade, their performance and efficiency decline, often leading to increased CO_2 emissions. Worn-out systems require more energy or resources to function, resulting in higher carbon emissions. To explore this connection, we introduce a model that examines the relationship between system degradation and CO_2 emissions. Let $r(\cdot)$ be a positive-valued function representing the rate of CO_2 emissions during operation, which is considered a function of the system condition, specifically its degradation. Therefore, the cumulative carbon dioxide emission related to system operation up to time t can be written as $\int_0^t r(X_u)du$. Recognizing that maintenance interventions also contribute to CO_2 emissions, cumulative CO_2 released up to time t can be written as:

$$E(t) = \int_0^t r(X_u)du + q_i N_i(t) + q_p N_p(t) + q_c N_c(t),$$

where q_i, q_p, and q_c represent the quantity of CO_2 emissions produced by inspection, preventive replacement, and corrective replacement, respectively. This function is also a function of two decision variables M and T.

Motivated by the framework of minimizing long-run average costs per unit time, we aim to minimize the long-run average carbon emissions per unit time, defined as follows:

$$EE(\theta) = \lim_{t\to\infty} \frac{\mathbb{E}\left(E(t)\right)}{t}.$$

Similarly, based on renewal theory, we have:

$$EE(\theta) = \frac{\mathbb{E}(E(S_1))}{\mathbb{E}(S_1)}.$$

The expectations can be computed using a Monte Carlo simulation; however, we must first specify the function $r(\cdot)$. Here, we consider a simple linear function of the form, $r(x) = \beta x$, where β is a positive constant. This indicates a strong positive relationship between carbon emissions and the system's level of degradation.

With this consideration and using the law of total expectation, we have:

$$\begin{aligned} &\mathbb{E}\left(\int_0^{S_1} r(X_u)du\right) \\ &\quad = \mathbb{E}\left(\mathbb{E}\left(\int_0^{S_1} r(X_u)du | S_1 = s_1\right)\right) \\ &\quad = \frac{\alpha\beta}{2\lambda}\mathbb{E}\left(S_1^2\right), \end{aligned}$$

where the second equality holds due to Tonelli's theorem.

5. Simulation Study

For illustrative purposes, let us consider a system that deteriorates according to a gamma process with parameters $\alpha = 1$ and $\lambda = 0.2$. The failure threshold is $L = 150$. The emission-related constants are $e_i = 0.1, e_p = 7, e_c = 7$, and the carbon emission rate, β, is set to 0.01. Moreover, the maintenance cost configuration is $c_i = 0.2, c_p = 10, c_c = 20$, and $c_d = 4$.

Figure 1 displays the Pareto front. It shows that the lowest maintenance cost value corresponds to the highest carbon emission and vice versa. The red point, which is tagged as the closest point with coordinates $(0.4916, 0.8604)$, has the shortest Euclidean distance from the Utopia point with coordinates $(0.4475, 0.8377)$. The corresponding decision variables are $T = 7.3889$, and $M = 85.81268$.

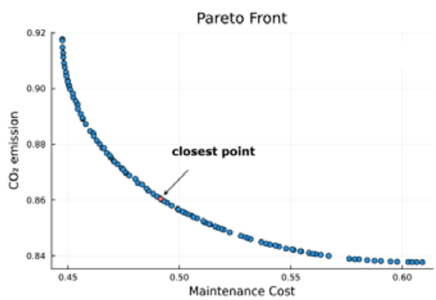

Fig. 1. Pareto front

The optimal values of each objective function at this compromise solution are then compared with those obtained from the corresponding mono-objective optimization models to evaluate performance differences. The optimal long-run expected maintenance cost is $EC^* = 0.447$. It is obtained for the decision variables $T = 9.265$ and $M = 78.571$. The cost function is given in Figure 2 with the closest point in red.

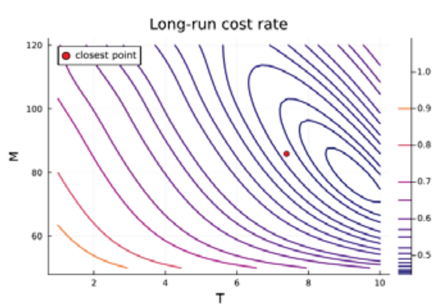

Fig. 2. Maintenance cost as a function of decision variables

The optimal long-run expected carbon emission is $EE^* = 0.8377$. This value is achieved with decision variables $T = 3.02$ and $M = 94.286$. The cost function is illustrated in Figure 3, where the closest point is marked with a red bullet.

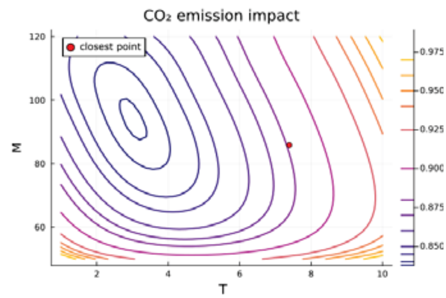

Fig. 3. Carbon emission as a function of decision variables

Shifting from an optimal maintenance cost configuration to a minimum carbon emission setup necessitates a substantial reduction in the time between inspections, that is, more intensive monitoring. This phenomenon is a consequence of the continuous increase of carbon emission rate as the system deteriorates, whereas there is no change in the preventive maintenance cost. In line with the reduction in the time between two consecutive inspections, the preventive maintenance threshold can be raised. According to Figures 2 and 3, the Euclidean distance to the closest point is shorter for the optimal maintenance cost scenario than for the optimal carbon emission scenario. The increase of cost and carbon emission from optimal to closest point values are respectively of 10% and 2.8%.

6. Conclusions

In conclusion, this study developed a stochastic modeling framework to represent industrial CO_2 emissions as a function of system degradation, capturing both deterministic operational factors and inherent randomness due to uncertain system conditions. By formulating cumulative emissions as a stochastic integral over time, the model enables the prediction of emission trajectories under degradation dynamics. Building on this framework, an optimal maintenance policy was derived to simultaneously minimize maintenance costs and cumulative CO_2 emissions. Simulation results validated the effectiveness of the proposed approach, demonstrating its capability to balance economic performance with environmental sustainability. For future studies, we will examine aperiodic maintenance policies using various stochastic processes capable of modeling non-decreasing degradation behavior. The objective will also be to derive closed-form expressions for the corresponding objective functions.

References

Bautista, L., I. T. Castro, and L. Landesa (2022). Condition-based maintenance for a system subject to multiple degradation processes with stochastic arrival intensity. *European Journal of Operational Research 302*(2), 560–574.

Certa, A., G. Galante, T. Lupo, and G. Passannanti (2011). Determination of pareto frontier in multi-objective maintenance optimization. *Reliability Engineering & System Safety 96*(7), 861–867.

Choi, J.-h. (2019). Strategy for reducing carbon dioxide emissions from maintenance and rehabilitation of highway pavement. *Journal of cleaner production 209*, 88–100.

De Almeida, A. T., R. J. P. Ferreira, and C. A. V. Cavalcante (2015). A review of the use of multicriteria and multi-objective models in maintenance and reliability. *IMA Journal of Management Mathematics 26*(3), 249–271.

De Weck, O. L. (2004). Multiobjective optimization: History and promise. In *Invited Keynote Paper, GL2-2, The Third China-Japan-Korea Joint Symposium on Optimization of Structural and Mechanical Systems, Kanazawa, Japan*, Volume 2, pp. 34.

Fang, G., R. Pan, and Y. Wang (2022). Inverse gaussian processes with correlated random effects for multivariate degradation modeling. *European Journal of Operational Research 300*(3), 1177–1193.

Levitin, G. (2006). *Computational intelligence in reliability engineering: evolutionary techniques in reliability analysis and optimization*, Volume 39. Springer.

Liu, B., J. Pang, H. Yang, and Y. Zhao (2024). Optimal condition-based maintenance policy for leased equipment considering hybrid preventive maintenance and periodic inspection. *Reliability Engineering & System Safety 242*, 109724.

Peng, J., Y. Yang, H. Bian, J. Zhang, and L. Wang (2022). Optimisation of maintenance strategy of deteriorating bridges considering sustainability criteria. *Structure and Infrastructure Engineering 18*(3), 395–411.

Wang, Y. and H. Pham (2011). A multi-objective optimization of imperfect preventive maintenance policy for dependent competing risk systems with hidden failure. *IEEE Transactions on Reliability 60*(4), 770–781.

Xu, A., J. Wang, Y. Tang, and P. Chen (2025). Efficient online estimation and remaining useful life prediction based on the inverse gaussian process. *Naval Research Logistics (NRL) 72*(3), 319–336.