

Incorporation of System Identification Based Ship Manoeuvring Model into Real-time Collision Risk Analysis

Cihad Celik

Liverpool Logistics, Offshore and Marine Research Institute, Liverpool John Moores University, Liverpool, UK. E-mail: C.Celik@2022.ljmu.ac.uk

Huanhuan Li

School of Engineering, University of Southampton, Southampton, UK. E-mail: Huanhuan.li@soton.ac.uk

Zaili Yang

Liverpool Logistics, Offshore and Marine Research Institute, Liverpool John Moores University, Liverpool, UK. E-mail: Z.Yang@ljmu.ac.uk

Collision risk assessment is a critical component of navigational safety, particularly in the context of emerging autonomous and semi-autonomous vessels. Although numerous collision risk models have been proposed, many existing approaches rely primarily on kinematic information derived from AIS data and commonly assume constant speed and straight-line motion for target ships (TS). Such assumptions limit the ability to capture realistic manoeuvring behaviour and introduce significant uncertainty in risk prediction. This study proposes an integrated framework that incorporates a data-driven ship manoeuvring model into real-time collision risk analysis. A system identification-based manoeuvring model, developed using a Dynamic Bayesian Network (DBN), is employed to predict future motion states, including position, speed, and heading. These predicted states are subsequently integrated into a DBN-based collision risk assessment model through CPA-based evaluation, enabling anticipatory risk prediction at one-minute intervals. Simulation studies are conducted using the KCS benchmark vessel to investigate both own ship (OS) and TS intention scenarios. The results demonstrate that the OS can quantitatively evaluate the impact of different rudder actions on collision risk and make informed, COLREG-compliant manoeuvring decisions in advance. Furthermore, the TS intention analysis highlights how considering simple control-related assumptions, such as rudder actions, can substantially influence future encounter geometry and collision risk evolution, even when such parameters are not directly available from AIS data. Overall, the proposed framework provides an intention-aware and uncertainty-informed collision risk assessment capability, offering valuable insights for the design of future autonomous navigation and decision-support systems.

Keywords: Collision risk assessment, Ship manoeuvring modelling, Dynamic Bayesian Networks, Intention analysis, Motion uncertainty, Maritime transportation safety.

1. Introduction

Ensuring navigational safety remains one of the most critical challenges in maritime transportation, particularly with the rapid advancement of autonomous and semi-autonomous vessel technologies. Although the International Regulations for Preventing Collisions at Sea (COLREG) (IMO 1972) provide a well-defined rule-based framework for collision avoidance, real-world accident statistics indicate that collisions continue to occur due to complex interactions among technical limitations, environmental disturbances, and human decision-making

processes. These challenges have motivated the development of quantitative collision risk assessment methods capable of supporting early and informed navigational decisions.

Over the past two decades, numerous collision risk assessment approaches have been proposed, including geometric indicators, ship domain concepts, velocity obstacle methods, probabilistic causation models, and hybrid frameworks combining multiple risk metrics (J. Liu et al. 2024; Jiang et al. 2024; He et al. 2024; Z. Liu et al. 2023). With the widespread adoption of Automatic Identification System (AIS) data, many of these methods have been implemented in real-time settings using vessel

position, speed, and course information, significantly improving encounter monitoring and situational awareness (Gil 2021; Z. Liu et al. 2023).

Despite these advances, a critical limitation remains in the treatment of target ship (TS) behaviour. In micro-scale ship encounter scenarios, TS motion is commonly assumed to follow constant speed and straight-line trajectories, even when sophisticated manoeuvring models are employed for the own ship (OS) (Vagale et al. 2021). While such assumptions simplify collision risk evaluation, they do not reflect realistic operational conditions and introduce substantial uncertainty into risk prediction. Recent studies have emphasised that uncertainty in TS manoeuvring behaviour represents one of the dominant sources of error in collision risk estimation (Zhang et al. 2025; Xin et al. 2023).

Developing high-fidelity physics-based manoeuvring models for multiple surrounding vessels is generally impractical due to the extensive experimental and numerical effort required. As a result, data-driven manoeuvring models have attracted increasing attention, as they can capture ship dynamics directly from data without relying on predefined hydrodynamic coefficients (Guo et al. 2025; Gil et al. 2025). In this context, probabilistic modelling techniques, particularly Dynamic Bayesian Networks (DBN), offer a suitable framework for representing temporal dependencies and uncertainty in ship motion prediction.

Motivated by these observations, this study proposes an integrated framework that combines a data-driven ship manoeuvring model with a DBN-based collision risk assessment approach. Rather than assuming deterministic TS behaviour, the framework investigates how incorporating manoeuvring-level information into predictive modelling can enhance intention-aware collision risk evaluation. By integrating predicted motion states into collision risk assessment, forward-looking and uncertainty-informed risk indicators are generated to support proactive decision-making in dynamic ship encounter scenarios.

The main contributions of this study are threefold:

- (i) the integration of data-driven manoeuvring prediction with real-time collision risk assessment;
- (ii) an explicit investigation of OS and TS intention effects on collision risk evolution; and
- (iii) a demonstration of how manoeuvring-level considerations can improve the robustness of collision risk estimation beyond AIS-based kinematic approaches.

2. Methodology

This section provides an overview of the models developed as data-driven ship manoeuvring in Section 2.1 and collision risk in Section 2.2. Subsequently, Section 2.3 presents and explains the integration strategy through which the developed models are combined into a unified framework. The key novelty of this chapter lies in the integrated application of the data-driven ship manoeuvring model and collision risk assessment.

2.1. Ship Manoeuvring Model

In this section, the manoeuvring problem is first described in Section 2.1.1. Subsequently, the system identification algorithm is introduced in Section 2.1.2 to train a model using the dataset presented earlier.

2.1.1. Problem Description

In calm-water conditions, ship manoeuvring motions are typically modelled using 3-DoF, corresponding to surge, sway, and yaw motions. Fig. 1 illustrates the two coordinate systems employed: the earth-fixed frame (x_0, y_0, z_0) and the body-fixed frame (x, y, z) . In the body-fixed frame at the midship, u and v denote the surge (forward) and sway (lateral) speeds, respectively, and r is the yaw rate.

The control input δ is the rudder angle; β is the ship's drift angle; and ψ is the ship's heading angle. G denotes the ship's centre of gravity.

The ship's manoeuvring dynamics are formulated in the body-fixed coordinate frame, as given by Eq. (1) (Abkowitz 1964).

$$\begin{cases} m(\dot{u} - vr - x_G r^2) = X \\ m(\dot{v} + ur + x_G \dot{r}) = Y \\ I_{zz} \dot{r} + mx_G(\dot{v} + ur) = N \end{cases} \quad (1)$$

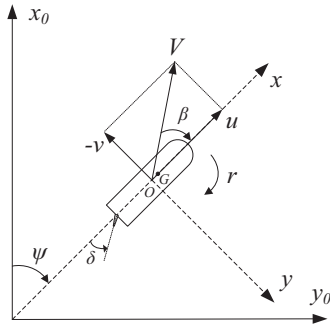


Fig. 1. Description of coordinate systems for ship manoeuvring motions.

Here, m denotes the ship's mass, I_{ZZ} is the moment of inertia about the z -axis, and x_G expresses the longitudinal position of the centre of gravity in the body-fixed frame. X and Y are the longitudinal and lateral forces acting on the hull, respectively, N indicates the yaw moment. Following Abkowitz (1964), and as formulated in Eq. (2), the hydrodynamic forces and moments acting on the ship are represented as nonlinear functions of the velocity components and the rudder angle.

$$\begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_G - Y_{\dot{r}} \\ 0 & mx_G - N_{\dot{v}} & I_{ZZ} - N_{\dot{r}} \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} f_x(u, v, r, \delta) \\ f_y(u, v, r, \delta) \\ f_z(u, v, r, \delta) \end{bmatrix} \quad (2)$$

Here, $X_{\dot{u}}$, $Y_{\dot{v}}$, $Y_{\dot{r}}$, $N_{\dot{v}}$, $N_{\dot{r}}$ represent the inertial hydrodynamic derivatives, while f_x , f_y , and f_z are nonlinear functions of u , v , r , and δ .

After rearranging the equation so that the acceleration terms remain on the left-hand side, the ship-dependent matrices are collected on the right-hand side. The right-hand side can then be written as nonlinear functions of the motion and control variables to be regressed against the acceleration terms as in Eq. (3).

$$\begin{cases} \dot{u} = s_x(u, v, r, \delta) \\ \dot{v} = s_y(u, v, r, \delta) \\ \dot{r} = s_z(u, v, r, \delta) \end{cases} \quad (3)$$

In conventional black-box modelling, the motion and control variables are treated as inputs, $\mathbf{X}_C = [u, v, r, \delta]^T$, while the acceleration terms are treated as outputs, $\mathbf{Y}_C = [\dot{u}, \dot{v}, \dot{r}]^T$. This formulation typically ignores the dependencies

among the input variables themselves. Yet that S_x , S_y , and S_z are nonlinear functions of u , v , r , and δ . Accordingly, analysing the mutual information shared among the motion and control variables is well aligned with the nature of the problem. A Bayesian reasoning framework is particularly suitable for this purpose, as it can encode these dependencies and, in turn, provide principled predictions of the acceleration terms.

2.1.2. System Identification Algorithm

The manoeuvring model is formulated using a Dynamic Bayesian Network (DBN) framework and is developed as a purely non-parametric, data-driven system identification model, without relying on predefined hydrodynamic coefficients or physics-based assumptions. The model is trained using free-running experimental data of the KCS benchmark obtained from the SIMMAN 2020 workshop (SIMMAN 2020). By directly learning the probabilistic and temporal dependencies among the motion states and control input, the DBN provides an interpretable yet flexible representation of ship manoeuvring dynamics.

In modelling, a Bayesian Networks (BN) is a Directed Acyclic Graphs ($\mathcal{G}$) which scores the conditional dependencies between a set of variables $X = \{X_1, \dots, X_N\}$. The joint distribution can be written as Eq. (4) below (Scutari 2010).

$$P(X) = \prod_{i=1}^N P(X_i | \pi(X_i)) \quad (4)$$

where $\pi(X_i)$ represents the parent set of X_i . Each node in the graph $\mathcal{G}$ represents a variable in X , while the arcs in $\mathcal{G}$ specify the dependencies among these variables. When all variables are continuous, as in the case of acceleration terms, velocity components in the dataset, it is common to assume that X follows a multivariate normal distribution. Consequently, the local distributions $P(X_i | \pi(X_i))$ take the form of linear regression models, where the node X_i serves as the response variable and its parent nodes $\pi(X_i)$ act as predictors (Scutari et al. 2024).

$$X_i = \mu_i + \pi(X_i)\beta_i + \varepsilon_i \quad (5)$$

Here, the μ_i denotes the intercept term in the local linear regression, capturing the mean of X_i in the absence of parental influence, the coefficient β_i represents the regression parameter associated with the parent nodes $\pi(X_i)$, while the error terms ε_i are assumed to be independent and normally

distributed with variance σ_i^2 , specific to each variable X_i . In cases where a node has no parents, the corresponding equation reduces to a model containing only the intercept and the error term. This static formulation of BN can be expanded in temporal space to obtain the dynamic interpretation as a Gaussian DBN. The learning process of a DBN involves two main stages: structure learning and parameter learning.

2.2. Collision Risk Model

This study adopts the collision risk assessment framework previously developed and validated by (Çelik et al. 2026). The same probabilistic collision risk model is directly incorporated into the present integrated system without structural modification, ensuring methodological consistency and allowing the focus to be placed on model integration and autonomous control performance rather than re-derivation of the risk formulation. The collision risk model is based on a discrete DBN framework, which enables probabilistic reasoning under uncertainty by explicitly modelling temporal dependencies among ship motion states, encounter geometry, and behavioural intentions of both the OS and TS. Detailed descriptions of the model structure, variable definitions, conditional dependencies, and validation results can be found in the original publication, to which the reader is referred for completeness.

2.3. Integration Strategy of the Models

Ship manoeuvring models constitute an essential component of autonomous navigation in the maritime environment. Although a wide range of approaches, based on experimental, numerical, and data-driven techniques, have been proposed in the literature, integrating these methods remains crucial for developing more comprehensive solution strategies.

As highlighted earlier, one of the major challenges in ship-to-ship collision risk analysis is the uncertainty associated with the motion of the TS, even when advanced risk assessment methods are employed. To address this challenge, a data-driven manoeuvring model is integrated with a DBN-based collision risk prediction framework.

The manoeuvring and collision risk models operate at different temporal resolutions. While motion prediction is performed at higher frequencies, both models are synchronised to a common prediction interval of 1 minute to ensure consistency.

Scenario analyses are conducted to demonstrate real-time applicability. During vessel operation, a rudder execution point is defined, after which both ship motion and collision risk predictions are performed. This integrated framework provides valuable predictive information in advance, supporting decision-makers or autonomous decision-making systems.

Fig. 2 illustrates the integrated framework of the proposed methodology. The data-driven manoeuvring model provides predicted motion states, position states, and heading angle information, which are incorporated into the collision risk assessment through CPA-based evaluation. This integration enables consistent motion prediction and collision risk prediction within a unified real-time decision-support structure.

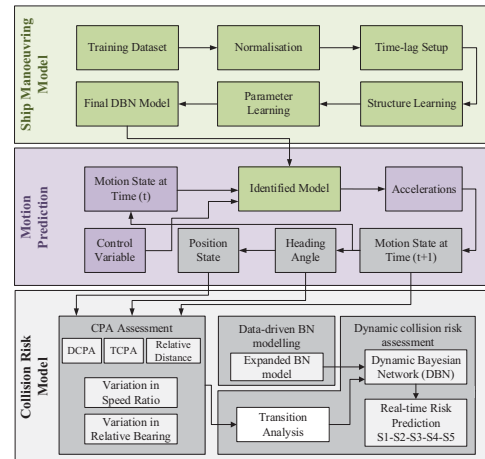


Fig. 2. Integrated data-driven manoeuvring model and DBN-based collision risk assessment framework

3. Simulation Results and Discussions

In the simulation studies, both the OS and the TS are assumed to be KCS vessels. The data-driven manoeuvring model is trained at a single operating condition corresponding to a Froude number of $Fr = 0.26$, which corresponds to 24 knots at full scale. To ensure consistency with the identified manoeuvring dynamics, the same forward speed is maintained across all simulation cases presented in this section.

3.1. Intention Analysis of the OS

In this scenario, the intention of the OS is analysed through a controlled collision case study. The initial positions of the OS and the TS

are configured such that a collision would occur at the 30th minute if no evasive action is taken. Both vessels follow predefined trajectories during this initial setup. Fig. 3 presents the trajectories of both vessels, where the position of each ship is shown at one-minute intervals. Based on these trajectories, the corresponding collision risk distribution over time is illustrated in Fig. 4. The results clearly indicate a progressive increase in collision risk, with the risk level reaching the *very high* category as the vessels approach the collision point at the 30th minute.

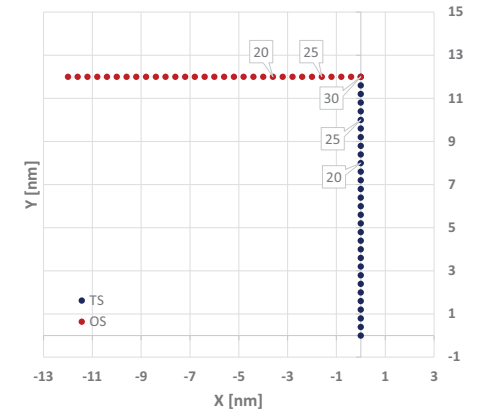


Fig. 3. Initial trajectories of the OS and TS at one-minute intervals without evasive manoeuvre.

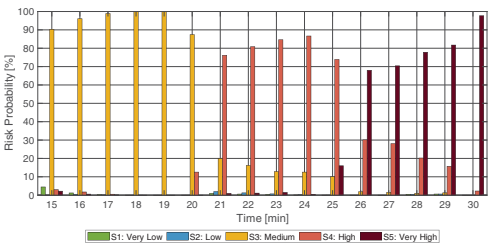


Fig. 4. Temporal distribution of collision risk levels for the baseline scenario leading to collision at the 30th minute.

An examination of the risk evolution reveals that, prior to the *high* risk level becoming dominant, the OS initiates a manoeuvre in accordance with COLREGs towards its starboard side. Specifically, at the 20th minute, the OS applies a rudder angle of 2 degrees in this scenario. The execution point of this manoeuvre is explicitly indicated in Fig. 5, together with the resulting deviation from the original straight-line trajectory. Following the rudder execution, the

data-driven manoeuvring model is employed to predict the OS motion at one-minute intervals. The predicted states include the vessel's position, speed, and heading angle. The resulting OS trajectory distribution, accounting for the applied rudder action, is also shown in Fig. 5. Although the scenario is simulated, the first 20 minutes are treated as if they correspond to real observed motion. From that point onward, the manoeuvring model is used to predict the future trajectory of the OS before the vessel physically reaches those locations. Using the predicted OS trajectory together with the associated motion states (speed and heading), the collision risk is re-evaluated.

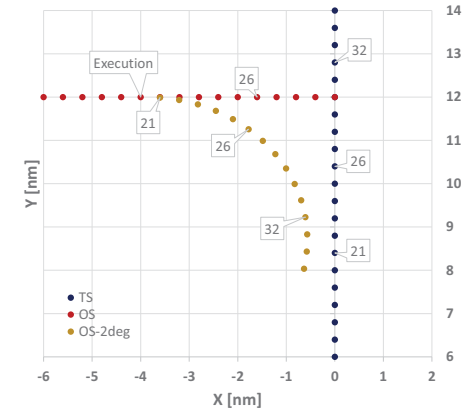


Fig. 5. Execution of COLREG-compliant manoeuvre and predicted OS trajectory using the data-driven manoeuvring model.

The updated collision risk distribution is presented in Fig. 6. In contrast to the baseline case shown in Fig. 4, where a collision occurs at the 30th minute with a *very high* risk level of approximately 98%, the early evasive manoeuvre significantly mitigates the risk. As a result, the *very high* collision risk level at the 30th minute is reduced to approximately 70%, followed by a gradual transition towards the *high* risk category.

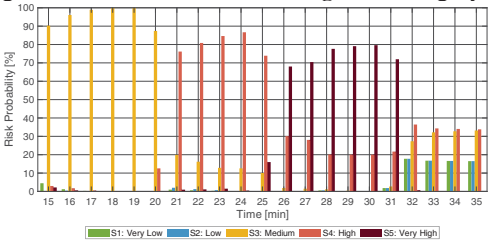


Fig. 6. Updated collision risk distribution after early manoeuvre based on predicted OS motion states.

3.2. Intention Analysis of the TS

In this scenario, the intention of the TS is analysed under a different operational assumption compared to the OS intention analysis. The initial trajectories of the OS and TS are configured such that no collision occurs throughout the encounter. Fig. 7 illustrates the corresponding OS and TS trajectories at one-minute intervals.

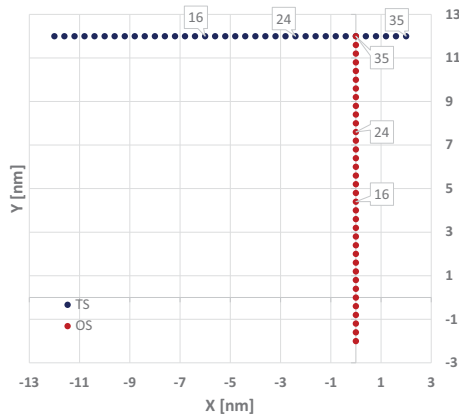


Fig. 7. Baseline trajectories of the OS and TS at one-minute intervals without collision.

Based on these trajectories, the collision risk distribution between the OS and TS is evaluated at one-minute intervals and presented in Fig. 8. Although no collision occurs, the results indicate elevated risk levels during the encounter. In particular, the *very high* collision risk level reaches approximately 40% at the 29th and 30th minutes, after which the risk decreases and transitions to the *high* level.

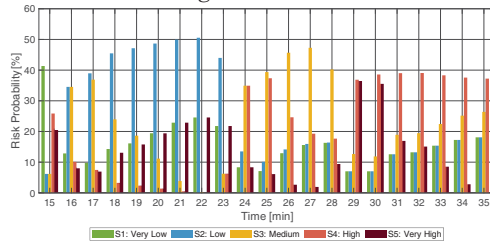


Fig. 8. Collision risk distribution over time for the baseline encounter without manoeuvre.

In this case, the OS is assumed to maintain a straight course at a constant speed. However, a sudden intention change of the TS is considered, where the TS executes an evasive or unexpected rudder action. Based on the instantaneous rudder angle and the TS forward speed at the execution moment, the data-driven manoeuvring model is

employed to predict the future trajectories of the TS. Fig. 9 presents the predicted OS and TS trajectories corresponding to rudder angles of 2 and 4 degrees executed by the TS at the 25th minute. The resulting trajectory distributions clearly demonstrate how different TS manoeuvring intentions lead to distinct encounter geometries, despite the OS maintaining its original course and speed.

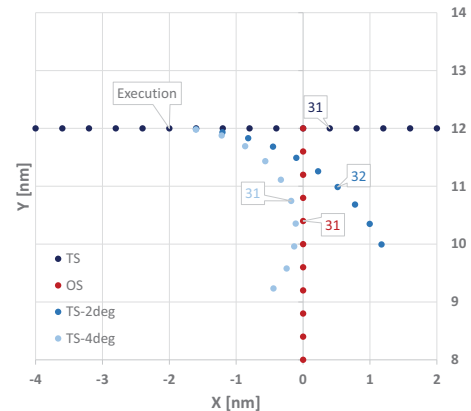


Fig. 9. Predicted OS and TS trajectories for sudden TS rudder actions at the 25th minute.

Using the predicted TS trajectories, the collision risk is re-evaluated from the perspective of the OS. The resulting collision risk distributions corresponding to the TS rudder actions are shown in Fig. 10 and 11. The results indicate that the *very high* collision risk level becomes particularly pronounced between the 31st and 32nd minutes, coinciding with the minimum relative distance between the vessels. This highlights the sensitivity of collision risk to sudden TS intention changes and underscores the importance of intention-aware prediction in dynamic encounter situations.

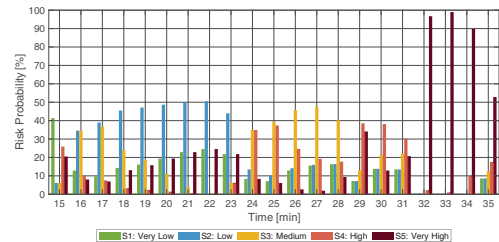


Fig. 10. Collision risk distribution corresponding to a 2° TS rudder manoeuvre.

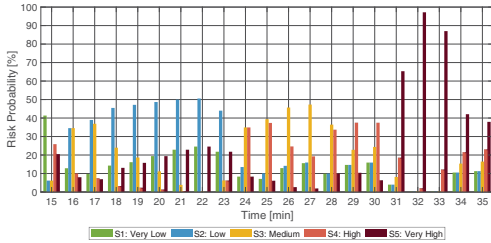


Fig. 11. Collision risk distribution corresponding to a 4° TS rudder manoeuvre.

3.3. Discussions

The intention analyses of the OS and TS collectively demonstrate the effectiveness of the proposed framework in supporting proactive and informed collision avoidance decisions under uncertainty. By integrating manoeuvring prediction with collision risk assessment, the OS is able to evaluate, in advance, how different rudder actions influence the evolution of collision risk and to determine the extent to which a potential collision can be mitigated.

In the OS intention analysis, the results indicate that the OS can initiate an early and COLREG-compliant rudder action based on the predicted collision risk distribution. By linking specific rudder angles to their corresponding reductions in collision risk, the framework enables the OS to make explicit and quantitative decisions regarding the magnitude and timing of evasive manoeuvres. This capability is particularly important, as it allows the OS to balance safety and manoeuvring efficiency without relying on reactive or last-moment actions.

The TS intention analysis further highlights the importance of accounting for uncertainty in the future motion of surrounding vessels. A potential concern may arise regarding the availability of TS manoeuvring inputs, such as rudder angle, which are not directly provided by standard AIS data. While AIS offers essential information, including position, speed, and course, it does not explicitly convey control-level parameters. However, the proposed framework highlights this limitation. By incorporating such parameters into a predictive modelling environment provides significant understanding of the TS motion. By exploring plausible TS manoeuvring actions through the data-driven manoeuvring model, the system effectively reduces motion uncertainty by generating multiple physically consistent TS trajectory hypotheses.

This approach allows the OS to assess collision risk not only under nominal encounter conditions but also under sudden or unexpected TS intention changes. The results show that even moderate TS rudder actions can significantly alter relative configuration and collision risk levels, particularly around critical time intervals where the relative distance between vessels becomes minimal. By explicitly accounting for these potential intention variations, the proposed method enhances the robustness of collision risk prediction beyond what can be achieved using AIS-based kinematic information alone.

Overall, the combined intention analyses demonstrate that the proposed framework enables the OS to anticipate and quantify the safety implications of both its own manoeuvring decisions and potential TS actions. By integrating manoeuvring-level parameters into the collision risk assessment process, the framework provides a powerful and flexible decision-support capability that effectively mitigates uncertainty in dynamic ship encounter scenarios.

4. Conclusions

This study has presented an integrated framework that incorporates a data-driven ship manoeuvring model into real-time collision risk analysis for ship-to-ship encounter scenarios. By coupling system identification-based motion prediction with a DBN-based collision risk model, the proposed approach enables anticipatory and intention-aware collision avoidance under uncertainty.

The simulation results demonstrate that the OS can quantitatively evaluate the effectiveness of different rudder actions in advance and determine the extent to which a potential collision can be mitigated. In particular, the intention analysis of the OS shows that early and COLREG-compliant manoeuvres, guided by predicted risk evolution, can significantly reduce collision risk before critical encounter conditions are reached. This provides a clear decision-support capability, allowing the OS to balance safety and manoeuvring efficiency without relying on reactive or last-moment actions.

The intention analysis of the TS highlights a fundamental limitation of collision risk assessment based solely on AIS data. While AIS provides essential kinematic information, such as position, speed, and course, it does not include control-level parameters, including rudder angle. Rather than claiming to resolve this limitation, the present study demonstrates that incorporating even simple

control-related assumptions within a predictive manoeuvring framework can substantially reduce uncertainty in TS motion. The results underline the importance of considering such parameters in the design of future collision avoidance and decision-support systems, as they provide valuable insight into potential TS behaviour beyond kinematic extrapolation.

For future work, the proposed framework can be extended by integrating additional sources of control- and intention-related information to further enhance the robustness of collision risk prediction.

Acknowledgement

This work is supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (Grant Agreement No. 864724).

References

- Abkowitz, M. A. (1964). *Lectures on Ship Hydrodynamics-Steering and Manoeuvrability*. HY-5. <https://trid.trb.org/View/159100>.
- Çelik, C., H. Li, J. Liu, M. Bashir, L. Zou, and Z. Yang (2026). 'Integrating Geometric and Causation Probability Approaches into Dynamic Bayesian Networks for Real-Time Collision Risk Prediction'. *Transportation Research Part E: Logistics and Transportation Review* 205 (January): 104520. <https://doi.org/10.1016/j.tre.2025.104520>.
- Gil, M. (2021). 'A Concept of Critical Safety Area Applicable for an Obstacle-Avoidance Process for Manned and Autonomous Ships'. *Reliability Engineering & System Safety* 214 (October): 107806. <https://doi.org/10.1016/j.res.2021.107806>.
- Gil, M., J. Montewka, and P. Krata (2025). 'Predicting a Passenger Ship's Response during Evasive Maneuvers Using Bayesian Learning'. *Reliability Engineering & System Safety* 256 (April): 110765. <https://doi.org/10.1016/j.res.2024.110765>.
- Guo, S., S. Zhuang, J. Wang, X. Peng, and Y. Liu (2025). 'Deep Learning-Based Non-Parametric System Identification and Interpretability Analysis for Improving Ship Motion Prediction'. *Journal of Marine Science and Engineering* 13 (10): 2017.
- He, Z., C. Liu, X. Chu, W. Wu, M. Zheng, and D. Zhang (2024). 'Dynamic Domain-Based Collision Avoidance System for Autonomous Ships: Real Experiments in Coastal Waters'. *Expert Systems with Applications* 255 (December): 124805. <https://doi.org/10.1016/j.eswa.2024.124805>.
- IMO, COLREG. (1972). 'Convention on the International Regulations for Preventing Collisions at Sea'. *Equasis, The World Merchant Fleet in 2015*.
- Jiang, Z., L. Zhang, and W. Li (2024). 'A Machine Vision Method for the Evaluation of Ship-to-Ship Collision Risk'. *Heliyon* 10 (3). <https://doi.org/10.1016/j.heliyon.2024.e25105>.
- Liu, J., J. Zhang, Z. Yang, C. Wan, and M. Zhang (2024). 'A Novel Data-Driven Method of Ship Collision Risk Evolution Evaluation during Real Encounter Situations'. *Reliability Engineering & System Safety* 249 (September): 110228. <https://doi.org/10.1016/j.res.2024.110228>.
- Liu, Z., B. Zhang, M. Zhang, H. Wang, and X. Fu (2023). 'A Quantitative Method for the Analysis of Ship Collision Risk Using AIS Data'. *Ocean Engineering* 272 (March): 113906. <https://doi.org/10.1016/j.oceaneng.2023.113906>.
- Scutari, M. (2010). 'Learning Bayesian Networks with the Bnlearn R Package'. *Journal of Statistical Software* 35 (July): 1–22. <https://doi.org/10.18637/jss.v035.i03>.
- Scutari, M., D. Kerob, and S. Salah (2024). 'Inferring Skin–Brain–Skin Connections from Infodemiology Data Using Dynamic Bayesian Networks'. *Scientific Reports* 14 (1): 10266. <https://doi.org/10.1038/s41598-024-60937-3>.
- SIMMAN. (2020). 'The Third Workshop on Verification and Validation of Ship Manoeuvring Simulation Methods'. <https://simman2020.kr/>.
- Vagale, A., R. T. Bye, R. Oucheikh, O. L. Osen, and T. I. Fossen (2021). 'Path Planning and Collision Avoidance for Autonomous Surface Vehicles II: A Comparative Study of Algorithms'. *Journal of Marine Science and Technology* 26 (4): 1307–23.
- Xin, X., Z. Yang, K. Liu, J. Zhang, and X. Wu (2023). 'Multi-Stage and Multi-Topology Analysis of Ship Traffic Complexity for Probabilistic Collision Detection'. *Expert Systems with Applications* 213 (March): 118890. <https://doi.org/10.1016/j.eswa.2022.118890>.
- Zhang, Z., H. Zhao, J. Wang, and H. Wang (2025). 'Efficient Collaborative Collision Avoidance Algorithm Considering COLREGs in Complex Multi-Ship Encounter Scenarios'. *Applied Ocean Research* 163 (October): 104753. <https://doi.org/10.1016/j.apor.2025.104753>.