

A Stochastic Hybrid Systems Framework for Probabilistic Digital Failure Twin Modeling Considering Multiple Dependent Failure Processes

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While Digital Twins (DT) are increasingly utilized to simulate complex systems, most existing models focus exclusively on nominal behavior, overlooking the failure dynamics essential for reliability modeling. To address this limitation, we propose a hierarchical Digital Failure Twin (DFT) framework designed to model failure behaviors emerging from multiple dependent processes. At the component level, we employ a Stochastic Hybrid System (SHS) approach: stochastic differential equations represent continuous degradation, while discrete transitions capture abrupt failure events and their mutual interactions. These component-level behaviors are then integrated into a system-level digital twin to analyze the impact of component failures on overall system performance. To facilitate efficient simulation and reliability assessment, we propose a semi-analytical method based on conditional moment dynamics and first-order second-moment (FOSM) approximation. This SHS-based method provides a unified analytical framework for evaluating both gradual degradation and sudden shocks. We demonstrate the efficacy of this approach through a robotic arm case study, modeling the impacts of motor-level wear and catastrophic failures caused by shocks. Numerical results indicate that our framework effectively captures complex dependencies among failure modes, offering accurate reliability estimates with significantly reduced computational overhead compared to traditional simulation methods.

Keywords: Digital failure twin, Stochastic hybrid systems, multiple dependent failure process, robotic arm

1. Introduction

Digital Twins (DT) have emerged as a transformative technology in the era of Industry 4.0, bridging the gap between physical entities and their virtual counterparts. Originally conceptualized by Grieves (2014), the Digital Twin is defined as a virtual representation that mirrors the life cycle of a corresponding physical system. In recent years, DTs have gained widespread adoption across academia and industry, particularly in aerospace, manufacturing, and automotive sectors (Tao et al. (2018)). The primary utility of these systems has historically focused on the simulation of nominal behavior—optimizing production flows, monitoring operational parameters, and fa-

cilitating geometrical integration (Rasheed et al. (2020)). By leveraging real-time data synchronization, these traditional DTs excel at reflecting the "as-built" or "as-operated" status of a system under normal conditions. However, as systems become increasingly complex, the exclusive focus on nominal behavior is insufficient for comprehensive lifecycle management where modeling of failure behavior becomes crucial.

To address the need for safety and durability, research has begun to pivot towards integrating reliability engineering with Digital Twin technologies. This evolution aims to transition from purely monitoring nominal states to predicting failure behaviors, often referred to as Prognostics and Health Management (PHM). Recent

works have explored DTs for tracking degradation, such as fatigue crack growth or battery capacity loss Erkoyuncu et al. (2019) and diagnosing faults Mc Court et al. (2025); Du, Zeng, Anwer, and Barros (Du et al.). These "Digital Failure Twins" (DFT) attempt to update reliability estimates based on sensor data. Despite these advancements, significant gaps remain. Existing approaches often rely on simplified statistical models that treat failure modes in isolation. They frequently overlook the complex reality of *multiple dependent failure processes*—where continuous degradation (e.g., wear, insulation aging) interacts with discrete stochastic events (e.g., random shocks, sudden component fractures) Zeng et al. (2023). Furthermore, current models tend to be single-layered, lacking a hierarchical structure that mathematically links component-level physics to system-level functional failures.

In this paper, we propose a novel framework for probabilistic digital failure twin modeling that rigorously addresses these limitations. We adopt a hierarchical modeling approach to capture the complexity of dependent failure processes. On the component level, we employ a Stochastic Hybrid System (SHS) formulation, extending the work of Fan et al. (2017, 2018). The SHS enables a unified mathematical representation where continuous degradation is modeled via Stochastic Differential Equations (SDEs), while discrete state transitions capture abrupt failure events and their interactions with degradation. These component-level behaviors are then integrated into a system-level digital twin to analyze functional failures and predict reliability. To demonstrate the efficacy of this framework, we present a numerical case study of a robotic arm. We illustrate how the proposed SHS-based DFT can capture complex dependencies—such as the impact of motor degradation on trajectory accuracy—providing significantly more efficient reliability estimates compared to traditional models based on simulation.

2. Stochastic Hybrid System for Hierarchical Digital Failure Twin Modeling

In this section, we develop a Stochastic Hybrid Systems (SHS)-based approach to model the behavior of a system within a hierarchical digital failure twin framework. We propose a two-level modeling architecture, as shown in Fig. 1. On the component level, we model the multiple failure processes (including statistically dependent degradation processes and random shocks) using an SHS framework (Sect. 2.1). On the system level, we employ a digital twin model that simulates normal performance metrics. Key parameters within this system-level digital twin are subject to component-level degradation governed by the SHS. Integrating the SHS with the system-level digital twin allows for a comprehensive simulation of system-level failure behavior (see Sect. 2.2 for details). A significant advantage of the SHS-based framework is that it allows efficient evaluating the system dynamics and reliability semi-empirically. In Sect. 2.3, we present how to use the SHS framework for component-level analysis, and in Sect. 2.4, we present how to estimate the system reliability efficiently based on the results from the component-level SHS and a first order second moment (FOSM) approximation.

2.1. Multiple dependent failure modeling on the component level

To capture the complex dynamics of component degradation, we utilize a SHS (Fan et al., 2017). The state space of the component is defined by a combination of continuous and discrete states. Let $q(t) \in Q = \{1, 2, \dots, n\}$ denote the vector of discrete state variables and $\mathbf{x}(t) \in \mathbb{R}^l$ denote the vector of continuous state variables.

It is important to emphasize that both $q(t)$ and $\mathbf{x}(t)$ model the degradation process, but they represent different aspects of the component's health. The continuous vector $\mathbf{x}(t)$ represents physical degradation parameters (e.g., wear, fatigue crack length) that evolve continuously. The discrete vector $q(t)$ represents distinct degradation modes or "health states" (e.g., perfect functioning, minor damage, severe damage), while the final discrete

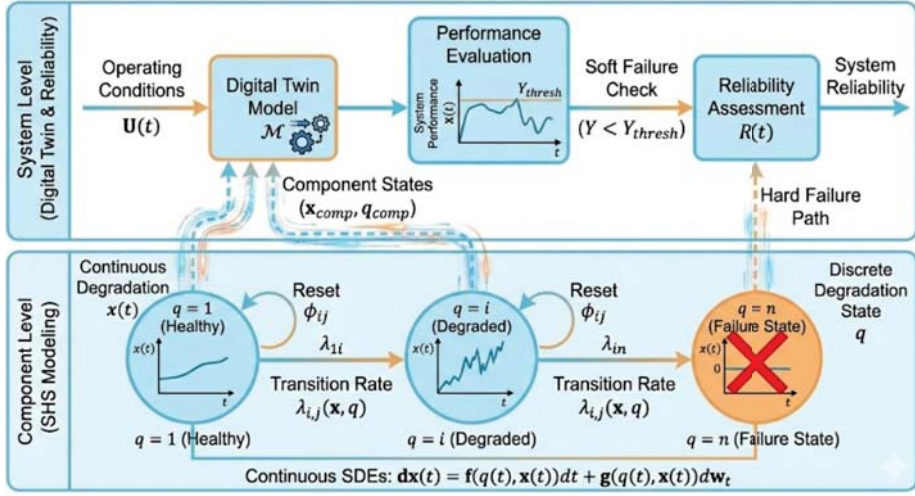


Fig. 1. A SHS-based framework for hierarchical digital failure twin modeling.

state, $q(t) = n$, represents the component failure state.

The SHS model is defined by the following dynamics:

- (1) Continuous Degradation: Within each discrete state $q(t)$, the evolution of the degradation parameters $\mathbf{x}(t)$ is governed by Stochastic Differential Equations (SDEs):

$$d\mathbf{x}(t) = \mathbf{f}(q(t), \mathbf{x}(t))dt + \mathbf{g}(q(t), \mathbf{x}(t))d\mathbf{w}_t \quad (1)$$

where $\mathbf{f} : Q \times \mathbb{R}^l \rightarrow \mathbb{R}^l$ describes the deterministic drift (degradation rate), $\mathbf{g} : Q \times \mathbb{R}^l \rightarrow \mathbb{R}^{l \times k}$ describes the diffusion intensity, and $\mathbf{w}_t$ is a k -dimensional Wiener process.

- (2) Discrete State Transitions: The component transitions between discrete degradation states $q(t)$ due to random shocks or accumulated damage. The transition from state i to state j occurs with a transition rate $\lambda_{ij}(q(t), \mathbf{x}(t))$.
- (3) Reset Maps: Upon a transition from state i to j , the continuous and discrete states are updated instantaneously via a reset map ϕ_{ij} :

$$(q(t), \mathbf{x}(t)) = \phi_{ij}(q(t^-), \mathbf{x}(t^-)) \quad (2)$$

This framework provides significant flexibility in modeling complex dependencies in failure pro-

cesses. Specifically, we can easily capture:

- (1) Dependencies between continuous processes: Through the coupling in the drift and diffusion functions $\mathbf{f}(q(t), \mathbf{x}(t))$.
- (2) Dependencies between discrete processes: By defining transition rates that depend on the current discrete state configuration.
- (3) Interaction between continuous and discrete processes: The transition rates λ_{ij} can depend on the accumulated physical degradation $\mathbf{x}(t)$ (e.g., a crack reaching a critical length triggers a state change), and conversely, the physical degradation rate $\mathbf{f}$ changes based on the discrete health state $q(t)$.

2.2. System-failure behavior modeling

The system-level behavior is modeled using a digital twin that takes the component status as input to predict system performance. Conceptually, the system digital twin is a mapping function $\mathcal{M}$:

$$Y(t) = \mathcal{M}(\mathbf{x}_{comp}(t), q_{comp}(t), \mathbf{U}(t)) \quad (3)$$

where $Y(t)$ represents the system performance metrics (e.g., power output), $\mathbf{U}(t)$ represents operating conditions, and $\{\mathbf{x}_{comp}, q_{comp}\}$ are the degradation states derived from the component SHS (see Sect. 3.1).

In this model, we can consider two types of system failures:

- (1) Catastrophic Failure (Hard Failure): If the components enter the complete failure state ($q_{comp}(t) = n$) (here we assume that there is no redundant components).
- (2) Soft Failure (Performance Failure): If the component is in a non-failure state, the degradation parameters $\mathbf{x}_{comp}(t)$ are injected into the digital twin $\mathcal{M}$. A soft failure occurs if the predicted system performance $Y(t)$ deviates beyond a failure threshold Y_{th} .

2.3. An efficient algorithm for component failure evaluation

To evaluate the component behavior, we derive the statistical moments of the degradation variables using the method of test functions (Fan et al., 2017). We define a test function $\psi_i^{(m)}(q, \mathbf{x})$ associated with the moment vector $\mathbf{m}$ while in discrete state i .

The time evolution of the expectation of this test function is governed by Dynkin's formula involving the extended generator $L\psi$. To accommodate the complexity of the SHS, the generator equation is expressed as (Fan et al., 2017):

$$(L\psi)(q, \mathbf{x}) = \frac{\partial \psi}{\partial \mathbf{x}} \mathbf{f}(q, \mathbf{x}) + \frac{1}{2} \text{trace} \left(\frac{\partial^2 \psi}{\partial \mathbf{x}^2} \mathbf{g}(q, \mathbf{x}) \mathbf{g}(q, \mathbf{x})^T \right) + \sum_{j \in Q} \lambda_{qj}(q, \mathbf{x}) (\psi(\phi_{qj}(q, \mathbf{x})) - \psi(q, \mathbf{x})) \quad (4)$$

Solving the resulting ODEs yields the conditional mean $\hat{\mu}_{\mathbf{x}|q=i}(t)$ and higher-order moments. This method is also capable of estimating the covariance between degradation processes. For example, by selecting a moment vector corresponding to the cross-term $E[x_a x_b | q = i]$, we can compute the covariance matrix $\Sigma_{\mathbf{x}|q=i}(t)$, which is essential for capturing the correlation between different degradation parameters.

2.4. System reliability assessment

The system reliability $R(t)$ is the probability that the system is free from both hard and soft failures.

$$R(t) = \sum_{i=1}^{n-1} \Pr(\mathbf{q}(t) = i) \cdot \Pr(\text{System Functional} | \mathbf{q}(t) = i) \quad (5)$$

For soft failure assessment, we must account for scenarios where the continuous degradation parameters $\mathbf{x}(t)$ are correlated. We evaluate the system performance $Y(t) = \mathcal{M}(\mathbf{x}(t))$ and define a limit state function $Z = Y(\mathbf{x}(t)) - Y_{th}$. Since $\mathcal{M}$ is generally non-linear, and the inputs $\mathbf{x}(t)$ are correlated random variables, we employ the First Order Second Moment (FOSM) method to approximate the reliability.

We linearize the performance function $Y(\mathbf{x})$ around the conditional mean vector $\hat{\mu}_{\mathbf{x}|q=i}$ using a Taylor series expansion:

$$Y(\mathbf{x}) \approx Y(\hat{\mu}_{\mathbf{x}|q=i}) + \nabla Y(\hat{\mu}_{\mathbf{x}|q=i})^T (\mathbf{x} - \hat{\mu}_{\mathbf{x}|q=i}) \quad (6)$$

where ∇Y is the gradient vector of the performance function.

Using this linear approximation, we estimate the conditional mean $\mu_{Y|i}$ and variance $\sigma_{Y|i}^2$ of the system performance. Crucially, the variance estimation must incorporate the correlation between inputs via the covariance matrix $\Sigma_{\mathbf{x}|q=i}$ derived in Section 3.3:

$$\begin{aligned} \mu_{Y|i}(t) &\approx Y(\hat{\mu}_{\mathbf{x}|q=i}(t)), \\ \sigma_{Y|i}^2(t) &\approx \nabla Y^T \Sigma_{\mathbf{x}|q=i}(t) \nabla Y \end{aligned} \quad (7)$$

The final system reliability is then estimated by summing the reliability contributions across all non-failure discrete states:

$$R(t) \approx \sum_{i=1}^{n-1} \mu_i^{(0)}(t) \cdot \Phi \left(\frac{Y_{th} - \mu_{Y|i}(t)}{\sigma_{Y|i}(t)} \right) \quad (8)$$

where $\mu_i^{(0)}(t)$ is the probability of being in discrete state i , and $\Phi(\cdot)$ is the standard normal cumulative distribution function.

3. Case Study: Digital Failure Twin Modeling of a Robotic Arm

3.1. Case study description

To demonstrate the proposed approach, we consider a hypothetical robotic arm used in precision applications. As shown in its schematic in Fig. 2, the robotic arm is composed of various components, including joints (motors), actuators, and structural frames. Its primary function is to control the end-effector to follow pre-defined trajectories with high accuracy. To simplify the analysis, we limit the desired function of the robot to be performing a required trajectory, which is a circle in a two-dimensional plane with radius of 1 (m) and centered in (2, 2) (m), as shown in Fig. 3 (a). Two failures are considered:

- (1) Hard failure: any motor that is in “complete failure” state.
- (2) Soft failure: the maximal euclidian distance between the actual end-effector position and the desired one, denoted by Y_e , is greater than a threshold value $Y_{th} = 0.015$ (m), as illustrated in Fig. 3 (b).

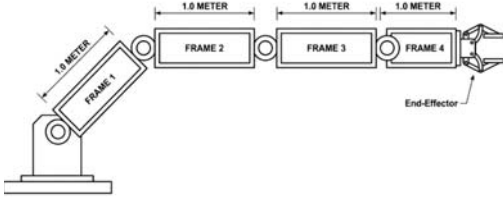


Fig. 2. A schematic of the robotic arm.

3.2. SHS modeling of component failures

We model the degradation of the robotic arm's joint actuators using a SHS. The state of the system is defined by a discrete mode $q(t) \in \{0, 1\}$ (where 1 = Alive, 0 = Hard failure due to traumatic shocks) and a continuous state vector $\mathbf{x}(t) = [x_1(t), \dots, x_4(t)]^T$ representing the friction coefficients of the four links, which are subject to degradation due to wears. We assume that the arrival of non-lethal shocks brings an

additional increment to the normal wear process, making these two failure processes statistically dependent, while lethal shocks cause a hard failure on all the four motors.

3.2.1. Continuous Degradation Dynamics

Let us assume that when the system is functioning ($q(t) = 1$), the degradation of each link i follows a SDE with logistic drift and diffusion:

$$dx_i(t) = rx_i(t) \left(1 - \frac{x_i(t)}{K_{cap}} \right) dt + \sigma_{diff} dW_i(t) \quad (9)$$

where r is the growth rate, K_{cap} is the saturation capacity, and σ_{diff} is the diffusion intensity. The Brownian motions $W_i(t)$ are independent across links, driving them apart over time.

3.2.2. Discrete State Transitions (Shocks)

The system is subject to common-cause shocks arriving at rate λ_{shock} . A shock affects *all* links simultaneously. If the shock magnitude W exceeds a threshold D_T , a hard failure occurs ($q(t) \rightarrow 0$). If $W \leq D_T$, a soft jump is incurred to the normal degradation process: all x_i increase by a random amount $J \sim \mathcal{N}(\mu_J, \sigma_J)$. This common shock induces correlation between the links.

3.2.3. Evaluation of Conditional Moments

To avoid computationally expensive Monte Carlo simulations, we derive the time evolution of the conditional moments. We define the unnormalized moments:

- $\pi_0(t) = P(q(t) = 1)$ (Survival Probability)
- $\pi_1^{(i)}(t) = \mathbb{E}[x_i(t) \mathbb{I}_{q(t)=1}]$ (First Moment Mass)
- $\pi_2^{(i)}(t) = \mathbb{E}[x_i(t)^2 \mathbb{I}_{q(t)=1}]$ (Second Moment Mass)
- $\pi_{cross}^{(ij)}(t) = \mathbb{E}[x_i(t)x_j(t) \mathbb{I}_{q(t)=1}]$ (Cross Moment Mass, $i \neq j$)

Using the extended generator of the SHS (see (Fan et al., 2017) for details), we derive the ODEs governing these moments. A critical challenge is the dependence on higher-order moments (e.g., $\mathbb{E}[x_i^3]$ and $\mathbb{E}[x_i x_j^2]$). We employ a log-normal moment closure technique:

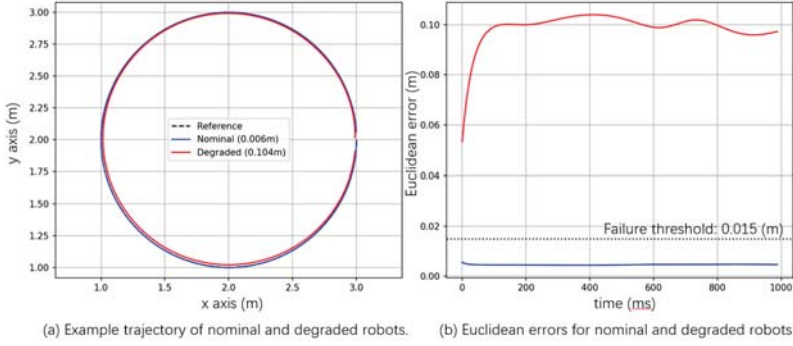


Fig. 3. Illustration of the robotic arm's desired and actual trajectory.

1. Single Moment Closure: We approximate $\mathbb{E}[x_i^3]$ assuming x_i follows a Log-Normal distribution:

$$\mathbb{E}[x_i^3] \approx \frac{\mathbb{E}[x_i^2]^3}{\mathbb{E}[x_i]^3} \quad (10)$$

2. Mixed Moment Closure (Correlation Correction): For the cross-term drift, we require $\mathbb{E}[x_i x_j^2]$. We use an improved closure that respects both independent and perfectly correlated boundaries:

$$\mathbb{E}[x_i x_j^2] \approx \frac{\mathbb{E}[x_i x_j] \mathbb{E}[x_j^2]}{\mathbb{E}[x_j]} \times \underbrace{\frac{\mathbb{E}[x_i x_j]}{\mathbb{E}[x_i] \mathbb{E}[x_j]}}_{\text{Correction Factor}} \quad (11)$$

This correction factor accounts for the high correlation induced by common shocks, preventing the overestimation of correlation decay.

After applying the moment closure techniques, we can eliminate the higher-order terms in the derived ODSs, which can be solved by ordinary solvers and used to derive the dynamics of conditional moments and covariance matrix.

3.3. System reliability assessment

To evaluate the probability of soft failure (performance violation), we reconstruct the joint probability distribution of the degradation vector $\mathbf{x}(t)$ using the moments derived from the SHS.

3.3.1. Surrogate Modeling of Performance

The robot fails if the tracking error $Y(\mathbf{x})$ exceeds a threshold Y_{th} . Since evaluating $Y(\mathbf{x})$ requires a physics simulation, we train a surrogate model $\mathcal{M}$

(Lookup Table) that maps the maximum friction level to error:

$$x_{crit} = \mathcal{M}^{-1}(Y_{th}) \quad (12)$$

This defines a critical surface: the system is safe if $x_i(t) < x_{crit}$ for all $i \in \{1..4\}$.

3.3.2. Multivariate Reliability Calculation

We assume the vector $\mathbf{x}(t)$ follows a Multivariate Log-Normal distribution. We construct the covariance matrix Σ_{LN} in the log-domain:

1. Recover Central Moments:

$$\begin{aligned} \mu(t) &= \frac{\pi_1(t)}{\pi_0(t)}, \\ \sigma^2(t) &= \frac{\pi_2(t)}{\pi_0(t)} - \mu(t)^2, \\ \text{Cov}_{ij}(t) &= \frac{\pi_{cross}(t)}{\pi_0(t)} - \mu(t)^2 \end{aligned} \quad (13)$$

2. Transform to Log-Parameters:

$$\sigma_{ln}^2 = \ln \left(1 + \frac{\sigma^2(t)}{\mu(t)^2} \right) \quad (14)$$

$$\mu_{ln} = \ln(\mu(t)) - \frac{1}{2} \sigma_{ln}^2 \quad (15)$$

$$\text{Cov}_{ln}^{ij} = \ln \left(1 + \frac{\text{Cov}_{ij}(t)}{\mu(t)^2} \right) \quad (16)$$

3. System Reliability Integration: The probability of soft failure is the probability that the system state vector falls outside the hyper-rectangular safe region defined by $\ln(x_{crit})$. We compute the reliability $R_{soft}(t)$ by integrating the Multivariate

Normal PDF over the safe region:

$$R_{soft}(t) = \int_{-\infty}^{\ln x_{crit}} \cdots \int_{-\infty}^{\ln x_{crit}} \phi_4(\mathbf{u}; \boldsymbol{\mu}_{ln}, \boldsymbol{\Sigma}_{LN}) d\mathbf{u} \quad (17)$$

Finally, the total system reliability is given by:

$$R_{sys}(t) = \pi_0(t) \cdot R_{soft}(t) \quad (18)$$

This approach rigorously accounts for the correlation between components, providing a precise estimation of system reliability compared to summing individual failure probabilities.

3.4. Results and discussions

The proposed SHS framework was validated against a high-fidelity Monte Carlo (MC) simulation. The results are shown in Fig. 4. Both methods were implemented in Python and executed on a personal computer equipped with an AMD Ryzen 7 4800H processor (2.9 GHz) and 16 GB of RAM.

As shown in Fig. 4, the SHS model accurately tracks the evolution of the system reliability metrics over the 3-year horizon. The probability of hard failure (due to traumatic shocks) predicted by the SHS matches the MC results almost perfectly, verifying the discrete state dynamics. The soft failure probability represents the likelihood that the robot's tracking error exceeds the critical threshold (0.015 m). The SHS method, utilizing the Multivariate Log-Normal integration over the safe region, provides a precise estimation of this risk. It successfully accounts for the "Series System" effect, where the high correlation between links implies that if one link degrades, others are likely degraded as well, affecting the joint probability of failure. The overall reliability as a result of both hard and soft failure is predicted in Fig. 4 (c). The SHS framework shows descend accuracy since the prediction almost matches the Monte Carlo simulation.

A significant advantage of the SHS framework is its computational efficiency. The Monte Carlo simulation, which involved simulating n_s individual robot samples over a long horizon (three years in this study). For each simulation run, the simulation horizon is discretized into small intervals and the digital twin model needs to be called at

each interval. The computational complexity of the traditional simulation methods are enormous, especially the simulation of digital twin is often time-consuming. In this study, a traditional Monte Carlo simulation of $n_s = 1000$ required approximately 45 minutes to complete. In contrast, the SHS analytical model, which involves solving a system of coupled Ordinary Differential Equations (ODEs) for the statistical moments. Even when the ODEs are complex and need to be solved numerically, it can be solved in a far more efficient manner. In our study, the analysis of the developed SHS approaches take only 2 minutes. This represents a speedup factor of over 20x, making the SHS approach highly suitable for iterative design optimization and real-time health monitoring where repeating expensive simulations is infeasible.

In conclusion, the results demonstrate that the SHS framework offers a powerful trade-off: it provides the accuracy required for safety-critical reliability assessment while delivering the computational efficiency necessary for modern digital twin applications.

4. Conclusions

In this paper, we have proposed a novel hierarchical framework for Probabilistic Digital Failure Twin (DFT) modeling, specifically designed to address the challenges of multiple dependent failure processes. By leveraging Stochastic Hybrid Systems (SHS), our approach provides a unified mathematical structure that rigorously captures the complex interactions between continuous physical degradation (e.g., wear) and discrete stochastic events (e.g., random shocks).

The proposed framework bridges the gap between component-level failure and system-level digital twin model that simulates nominal behaviors. Through the application of a robotic arm case study, we demonstrated the method's capability to model the propagation of component degradation—specifically joint friction—to system-level trajectory errors. The numerical results validated that the SHS-based analytical approach achieves high accuracy, matching the reliability estimates of high-fidelity Monte Carlo simulations. Cru-

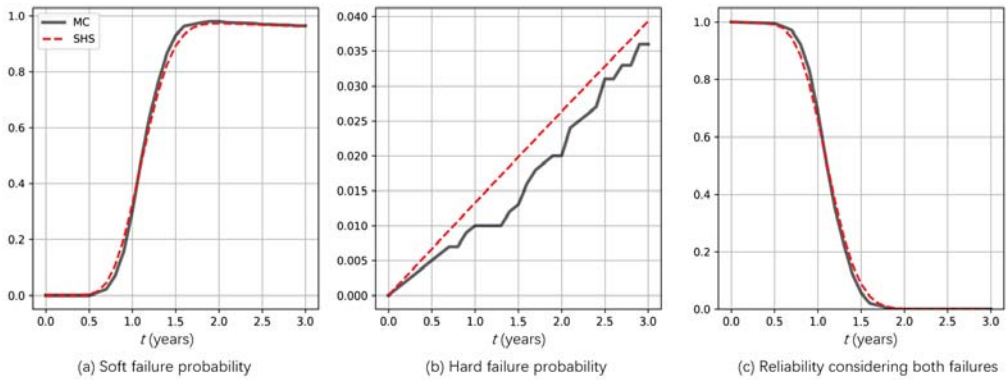


Fig. 4. A comparison between the SHS framework and Monte Carlo simulation.

cially, the proposed method offers significant computational advantages, reducing the processing time by an order of magnitude compared to traditional simulation methods. This efficiency is paramount for Digital Twin applications, where near real-time decision-making is required.

Future work will focus on two main directions: first, extending the framework to incorporate real-time sensor data for online Bayesian updating of the degradation parameters; and second, scaling the approach to model larger, multi-component systems with complex topological dependencies.

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