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A Comparative Multivariate Reliability Analysis Approaches Accelerated Failure Time (AFT) and Cox PH regression: Case in Oil & Gas Well

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Deep-offshore operations subject well components to extreme conditions where physical maintenance is often impossible. Given the limited number of wells per platform, high reliability and real-time risk monitoring are essential. Because retrieving components for inspection is difficult, failure analysis relies heavily on operational variables, requiring robust multivariate reliability assessments. This paper evaluates two primary methods for monitoring well integrity: the Cox Proportional Hazards (Cox PH) model and the Accelerated Failure Time (AFT) model. While Cox PH is a staple of survival analysis, AFT is more frequently utilized in engineering reliability. The study compares these approaches using a dataset of Down Hole Safety Valve (DHSV) lifetimes, comprising 550 observations from 448 wells. The analysis focuses on the impact of two critical stressors—temperature and pressure—on component longevity. By implementing goodness-of-fit tests and graphical reliability curve comparisons, the research highlights the advantages and disadvantages of each method. The results demonstrate that an integrated approach—combining the AFT method with a 2-parameter Weibull model—effectively generates reliability curves that capture the dynamic effects of temperature and pressure variations. This multivariate framework provides a superior tool for predicting well degradation and ensuring operational safety in challenging offshore environments.

Keywords: Multivariate Reliability Analysis, Cox regression, Proportional Hazard, Accelerated Failure Time, Well, Oil and Gas.

1. Introduction

Offshore operations at great depths subject well components to severe conditions. Operating at great depths, below the seabed, and performing repairs and maintenance where human physical presence is impossible, becomes even more complex. In this scenario, another challenge is reducing the number of wells per platform, which demands greater reliability and real-time risk monitoring.

Failure analysis of components is primarily based on operational variables, as special resources are needed to remove well components, and often, removal is unfeasible. Therefore, this context demands multivariate reliability assessments to monitor well integrity.

The Cox Proportional Hazard model is one of the most popular for handling covariates in healthcare. Originally, in Cox (1972), it provided a mathematical treatment dedicated to discrete variables, but applications have emerged that have extended its use, such as Jerzy K. Filus and Lidia Z. Filus (2017), which shows the method for constructing virtually any joint survival function given marginal survival functions.

Since Wei (1992) introduced the idea of accelerating or decelerating time as a way to predict the effect of other variables on the time to failure, a large number of scientific papers have been published on the subject.

These models can be based on probability distributions as initially proposed, or even use functions outside the context of distributions as in Pang et al. (2021) and Crowther et al. (2023). The first paper models a baseline hazard function by regression B-splines, allowing for the estimation of a variety of smooth and flexible shapes. The second paper concentrates on using restricted cubic splines to model the baseline to provide substantial flexibility.

Although it is possible to use extremely flexible functions without considering the physics involved in component failure, the use of established statistical models in failure modeling offers the advantages of decades of use and developed knowledge.

2. Methodology

This paper compares the assumptions required for each Cox PH model and the AFT method, conducts a graphical analysis of the reliability curve, and implements goodness-of-fit tests and metrics to evaluate the advantages and disadvantages of both approaches in analyzing the significance of the well degradation stressors, temperature, and pressure.

For the comparison process, the reliability analysis is applied to a dataset of Down Hole Safety Valve lifetimes, using Cox Regression Proportional Hazards (PH) and Accelerated Failure Time (AFT) methods. The dataset totals 448 wells with 550 observations. The attributes used were duration, event, temperature in the flow, and pressure in the flow.

For both approaches, Cox PH and Weibull AFT, the reliability curves were graphically compared.

The influence of temperature and pressure variations on reliability was graphically expressed.

3. Semi-parametric Cox Proportional Hazards Failure Rate Model

Proportional hazards models are a class of reliability (or survival, where such models are most commonly used) models. Survival models relate the time elapsed until an event occurs to one or more covariates that may be associated with that time interval. In the context of this analysis, the event is the failure of the DHSV valve.

In a proportional hazards model, the single effect of a unit increase in a covariate is multiplicative with respect to the failure rate (or hazard, as it is called in healthcare and related fields). The failure rate at time t is the short-term probability dt that an event will occur between t and $t + dt$, given that no event has yet occurred up to time t. In mathematical terms, $h(t) = f(t)/R(t)$, where $f(t)$ is the probability density function and $R(t)$ is the reliability function up to time t.

Other types of reliability models, such as accelerated failure time models, do not exhibit proportional failure rates. The accelerated failure time model describes a situation where the mechanical history of an event is accelerated (or decelerated) as a function of covariates.

Proportional failure rate models are composed of two parts: (i) the underlying basis failure rate function, denoted $h_0(t)$. This function describes how the failure rate changes over time at the basis levels of covariates; and (ii) the effect parameters, which describe how the failure rate varies in response to explanatory covariates.

The condition of proportional failure rates assumes a multiplicative relationship between the covariates and the failure rate value. In the simplest case of stationary coefficients, for example, using a more resistant material can halve the failure rate for any instant t, while the base failure rate can vary. Note that this does not double the lifespan of the item under study. The effect of the covariates on the lifespan of each item depends on the shape of $h_0(t)$. The covariates represent only binary predictors (those that include or exclude a given category, such as manufacturer). In the case of a continuous covariate x, it is usually assumed that the failure rate responds exponentially. For each unit increase in x, a proportional increase in the failure rate of $k = \exp(x)$ results.

According to Cox (1972), if the assumption of proportional failure rates is maintained, it is possible to estimate the effect parameters, denoted by β_i, without any consideration of the failure rate function. This approach is widely applied to survival data and in this context is called the Cox proportional hazards model. In reliability engineering, the word hazard means failure rate.

This method is commonly abbreviated to the Cox model or the proportional hazards model.

Let $X_i = (X_{i1}, \cdots, X_{ip})$ be the observed values of the p covariates of item i. The failure rate function for the Cox proportional failure rates model has the form:

$$h(t|X_i) = h_0(t) \exp\left(\beta_1 X_{i1} + \cdots + \beta_p X_{ip}\right). \tag{1}$$

Note that Eq. 1 can be written as $h(t|X_i) = h_0(t) \exp\left(X_i \centerdot \beta\right)$ where $X_i \centerdot \beta$ is the dot product between the vectors $X_i = (X_{i1}, \cdots, X_{ip})$ and $\beta = (\beta_1, \cdots, \beta_p)$

This expression provides the failure rate function at time t for the item i with the covariates vector (explanatory variables) X_i . Note that among the items analyzed, the base failure rate $h_0(t)$ is the same (it has no dependence on i). The influence on item failure rates is expressed by the base scaling factor $\exp\left(X_i \centerdot \beta\right)$

Assume there is only one covariate x, and therefore a single coefficient β_1. Consider the effect of increasing x by 1 unit:

$$\begin{aligned} h(t|x+1) = & \quad h_0(t) \exp\left(\beta_1 (x+1)\right) \\ = & \quad h_0(t) \exp\left(\beta_1 x + \beta_1\right) \\ = & \left[h_0(t) \exp\left(\beta_1 x\right)\right] \exp\left(\beta_1\right) \\ = & \quad h(t|x) \exp\left(\beta_1\right) \end{aligned} \tag{2}$$

Increasing a covariate by 1 unit modifies the original failure rate by the constant $\exp\left(\beta_1\right)$. Rearranging, we have:

$$\frac{h(t|x+1)}{h(t|x)} = \exp\left(\beta_1\right). \tag{3}$$

The right-hand side of the equation 3 is constant over time.

Consider two items with covariates X_i and X_j. The ratio between their failure rates is:

$$\begin{aligned} \frac{h(t|X_i)}{h(t|X_j)} = & \quad \frac{h_0(t) \exp(X_i \centerdot \beta)}{h_0(t) \exp(X_j \centerdot \beta)} \\ = & \quad \frac{\exp(X_i \centerdot \beta)}{\exp(X_j \centerdot \beta)} \\ = & \exp\left((X_i - X_j) \centerdot \beta\right) \end{aligned} \tag{4}$$

The ratio between the failure rates of two items is constant, so the failure rates are said to be proportional. Most high-level programming languages offer libraries to calculate the coefficients β_i. The mathematical details of these calculations will not be covered in this article.

In the equation 1, the term $h_0(t)$ is the reference failure rate (the baseline rate) and the term $\exp\left(X_i \centerdot \beta\right)$ is known as the partial failure rate. The partial failure rate increases or decreases the failure rate based on the covariates.

4. Weibull Accelerated Failure Time Model

Assume that the time to failure is the random variable $\mathbb{T}$. The accelerated failure time method assumes that time can be sped up or slowed down by means of m covariates X. Thus, if the number of times to failure is n, and $i = 1, 2, \cdots, n$, each time $\mathbb{T}_i$ is expressed by:

$$\ln \mathbb{T}_i = B_0 + B_1 X_{1,i} + B_2 X_{2,i} + \cdots + B_m X_{m,i} + S\epsilon_i, \tag{5}$$

where B_1, B_2, $\cdots$, B_m are unknown scalar coefficients of the covariates, $X_{1,i}$, $X_{2,i}$, $\cdots$, $X_{m,i}$ are the i-th known values of the covariates 1, 2, up to m, S is an unknown scalar constant and ϵ_i, are independent and identically distributed random variables (i.i.d), whose common distribution function is not specified (Wei (1992)). In other words, it is random noise following a probability distribution.

In Equation 5, the function $\ln(\bullet)$ can be replaced by any other function $j(\bullet)$ defined for the intervals of values and monotonically increasing (Wei (1992)).

Organizing into two parts that separate covariates from noise, or, in other words, adopting $\mu_i = B_0 + B_1 X_{1,i} + B_2 X_{2,i} + \cdots + B_m X_{m,i}$, we have $\ln\left(\mathbb{T}_i\right) = \mu_i + S\epsilon_i$. Isolating each time $\mathbb{T}_i$ we get:

$$\mathbb{T}_i = \exp\left(\mu_i + S\epsilon_i\right) \tag{6}$$

Once we know the statistical distribution of $\mathbb{T}$, or that is, its density function f_T and its cumulative distribution function F_T, the statistical distribution of ϵ needs to be compatible, with density function g and cumulative distribution function G.

Since $j(\bullet)$ is a monotone increasing function, then:

$$P(\mathbb{T} \leq t) = P\left(\epsilon \leq \frac{j(t) - \mu}{S}\right), \qquad (7)$$

and consequently:

$$P(\mathbb{T} > t) = P\left(\epsilon > \frac{j(t) - \mu}{S}\right). \qquad (8)$$

Most high-level programming languages have specialized libraries for AFT models for some statistical distributions, including the Weibull distribution. This article will not detail the necessary mathematical deductions or the use of such libraries.

5. Results

The same data were used in both analyses. There are 142 observations, 31 observed events, and 111 censored times. All observations have associated pressure and temperature values. Log-likelihood, concordance index, and AIC (Akaike Information Criterion) (see Akaike (1974)) values were calculated for the Cox PH and the Weibull AFT models.

5.1. *Cox PH*

Table 1 shows the calculated log-likelihood, concordance index, and AIC values for the Cox PH model.

Table 1. *

Metrics of Cox PH fitness.

Metric	Value
log_likelihood_	-110.25
concordance_index_	0.79
AIC_	224.50

The calculation of the coefficients is performed by maximizing the log-likelihood. In this example, log-likelihood = -110.25.

A C-index value of 0.79 means that in 79% of the pairs of items compared, the one that the model indicated as having the highest failure rate (due to pressure and temperature) was, in fact, the one that would present the highest failure rate. Generally, values $0.70 <$ C-index < 0.80 mean that the model has good quality.

AIC (Akaike Information Criterion) is a metric used to compare different models applied to the same dataset. The value found for the Cox PH model is 224.50. The lower the AIC value, the better the model.

Table 2 shows the summary of the fit. The covariates, parameter names, coefficient values, and their exponentials, standard errors, z-statistics, and p-values are included.

Table 2. *

Summary of Cox PH fitness.

Covariate	Pressure in flow	Temperature in flow
Parameter	β_P	β_T
Coefficient	6.46E-06	2.45E-02
exp(coef)	1.00	1.02
se(coef)	8.86E-05	6.03E-03
z	7.29E-02	4.07
p	0.94	4.73E-05

The calculated coefficient values were $\beta_P = 6.46 \times 10^{-6} psi^{-1}$ and $\beta_T = 2.45 \times 10^{-2 \text{o}} C^{-1}$. The exponential values of the coefficients are $\exp(\beta_P) = 1.00$ and $\exp(\beta_T) = 1.02$. This means that when the pressure variable increases by one unit, the failure rate remains practically unchanged, and when the temperature increases by one unit, the failure rate increases by approximately 2%.

The standard error values for each coefficient of the Cox PH model are $\text{se}(\beta_P) = 8.86 \times 10^{-5}$ and $\text{se}(\beta_T) = 6.03 \times 10^{-3}$. Note that these values are slightly larger than those found in Table 6 for the calculation of the Weibull AFT model. If the standard error (in Cox PH model) for Pressure is high, the model has difficulty determining whether the increase in pressure actually consistently multiplies the failure rate.

The z-value measures how far the coefficient is from zero in units of standard error. The larger the absolute value of z (positive or negative), the more important that variable is to the model. Thus, the variable representing temperature has more influence on the model's responses. The values for this measurement for the pressure and temperature variables are $z_P = 7.29 \times 10^{-2}$ and $z_T = 4.07$

$(z_T > z_P)$. This shows that temperature has more influence than pressure on the variation of the failure rate.

The p-value is the tool that indicates whether the impact of a variable (pressure or temperature) is statistically significant or not. If the p-value $<$ 0.05, there is statistical evidence that the variable actually affects the failure rate. If the p-value $>$ 0.05, there is not enough evidence to say that the variable affects the failure rate. The effect may have been caused by a coincidence in the sample. The p-value for the pressure variable is greater than 5%. According to this result, pressure is not statistically significant, but temperature is.

The reliability and cumulative failure rate curves for pressures of 1,000 psi and 10,000 psi are shown in Table 3. Each graph presents the curves for temperatures of 10ºC, 20ºC, 40ºC, 60ºC, 80ºC, and 110ºC. The higher the temperature, the lower the reliability and the higher the cumulative failure rate for the same instant. The difference between the graphs with pressures of 1,000 psi and 10,000 psi is practically nonexistent; therefore, graphs with intermediate values were not plotted.

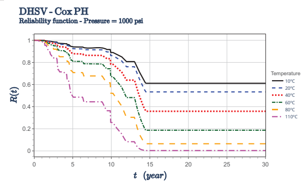
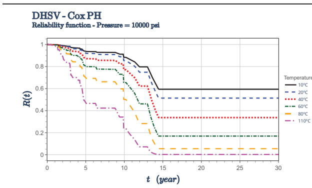
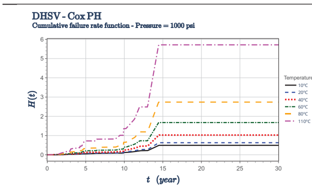
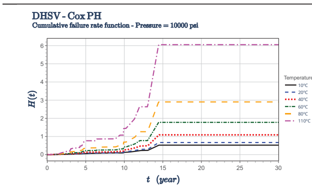
The cumulative failure rate was used because the instantaneous failure rate could not be used due to the steps (discontinuities) in the reliability values calculated non-parametrically with the Kaplan-Meier estimator (see Kaplan and Meier (1958)).

The curves in all the graphs stabilize after 15 years because there was no time to failure exceeding this time interval.

Table 4 presents the reliability and cumulative failure rate curves. Each graph was plotted with a single temperature value, but each curve represents different pressure values. Since pressure, in this Cox PH model, was not a statistically significant variable, the curves for each pressure are practically superimposed. The higher the temperature, the steeper the decay of the reliability function and the lower the value at which it becomes stable.

Table 3. *

Cox PH: reliability and accumulated failure rate curves.

Reliability function


Accumulated failure rate function



5.2. *Weibull AFT*

Table 5 presents log-likelihood, concordance index, and AIC values for the Weibull AFT.

The log-likelihood value is -98.90. This is slightly higher than that found for the Cox PH model, but it's important to remember that Cox PH is a semi-parametric model while Weibull AFT is parametric.

The C-index is 0.79, approximately the same as that found in Cox PH. This means that both fits are considered to be of good quality.

The AIC for the Weibull AFT model is 205.80. This value is slightly lower than that calculated for Cox PH. This result confirms the AFT model as better than the Cox PH model.

Table 6 shows the summary of the Weibull AFT

Table 4. *

Cox PH modeling: reliability function and accumulated failure rate curves for temperature values.

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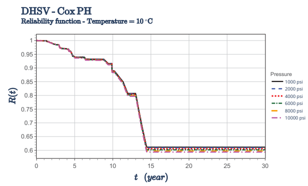
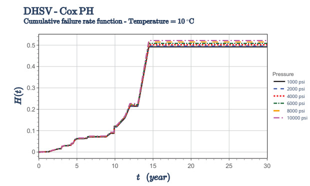
<tr><td>
</td><td>
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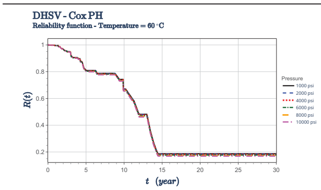
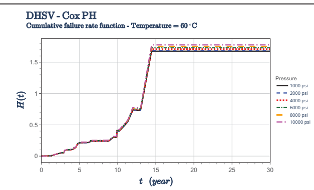
<tr><td>
</td><td>
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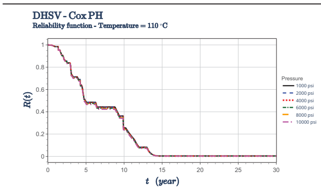
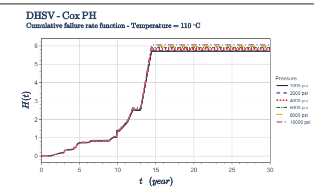
<tr><td>
</td><td>
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Table 5. *

Metrics of Weibull AFT fitness.

Metric	Value
log-likelihood_	-98.90
concordance index	0.79
AIC	205.80

fit. There are 4 columns, 2 of which relate to the covariates pressure and temperature, and the other 2 are the intersections. The first intersection is β_0, which makes up the vector β, and the second is s, the inverse of the exponent of the Weibull distribution (shape parameter).

The coefficients of each parameter are in the third row of the table. The exponentials of these values are in the fourth row and are $\exp(\beta_P) = 1.00$ and $\exp(\beta_T) = 0.97$. This means that increasing the pressure by one unit has practically no effect on the item's lifetime, and increasing the temperature by one unit reduces the item's lifetime by 3%.

The standard error values are in the fifth row. The larger the standard error, the less certainty the model provides that its variable reduces or increases the lifespan of an item. The standard error for temperature 4.92×10^{-3} is greater than the standard error for pressure 5.24×10^{-5}. This means we are less certain about the influence of temperature on increasing or decreasing lifespan.

On the other hand, the statistic z_T has a larger absolute value than z_P. This means that temperature affects lifespan more intensely than pressure does.

Finally, the p-value results show that there is weak evidence to state that pressure affects lifespan (p-value > 0.05), but the opposite is true for the temperature variable (p-value < 0.05).

The Weibull AFT is a parametric multivariate model. Unlike the Cox PH model, the Weibull AFT model allows plotting curves of the probability density function and the failure rate function. This is because this model deals with continuous variables, so the curves are not stepwise as in the Cox PH model. Table 7 shows these curves for pressure values of 1,000 psi and 10,000 psi. Note that the graphs for these pressures are practically identical.

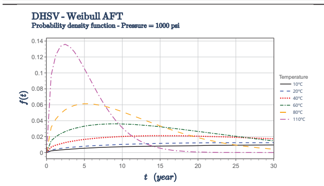
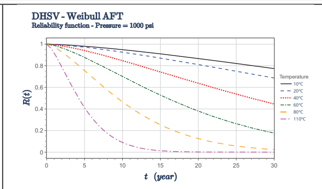
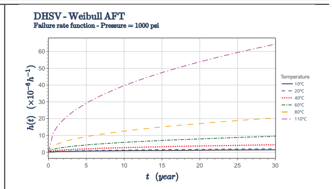
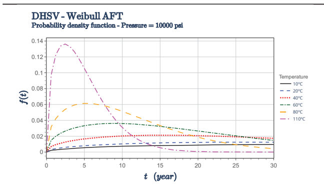
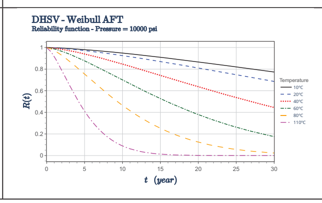
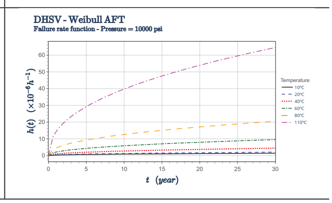
Table 6. *

Summary of Weibull AFT fitness.

Covariate	Pressure in flow	Temperature in flow	Intercept	Intercept
Parameter	β_P	β_T	β_0	s
Coefficient	-2.08E-07	-2.65E-02	4.61	0.69
exp(coef)	1.00	0.97	100.60	2.00
se(coef)	5.24E-05	4.92E-03	0.51	0.14
z	-3.97E-03	-5.40	9.08	4.98
p	9.97E-1	6.72E-08	1.12E-19	6.20E-07

Table 7. *

Probability density function, reliability, and failure rate curves for pressure values 1,000 psi and 10,000 psi.

Probability density function	Reliability function	Failure rate function
		
		

Each curve is plotted at a different temperature value. Higher temperatures result in lower reliabilities and higher failure rates for the same time to failure.

Table 8 presents the curves of the probability density, reliability, and failure rate functions for different temperature values. As the temperature increases, the reliability values decrease for the same time to failure. After 30 years at a temperature of 10 °C, the reliability is approximately 0.77. At a temperature of 110ºC, in 5 years the reliability has already reached 0.4. The failure rate shows the opposite behavior. The item has an instantaneous failure rate in 30 years close to $1.4 \times 10^{-6} h^{-1}$ while subjected to 10ºC, and at a temperature of 110ºC, this value would be greater than $60 \times 10^{-6} h^{-1}$ according to the Weibull AFT model.

6. Conclusions

This article compared the Cox PH and Weibull models in the context of DHSV valves installed in offshore oil wells.

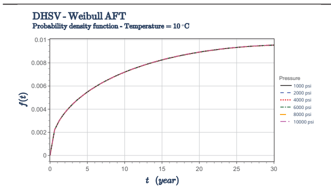
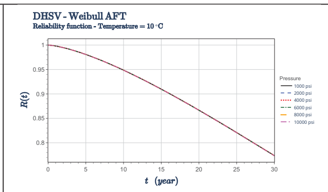
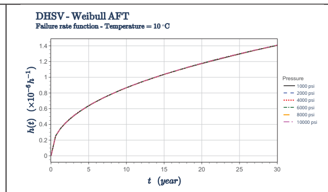
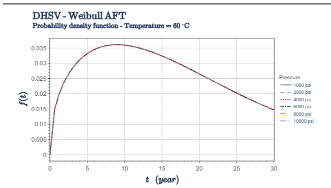
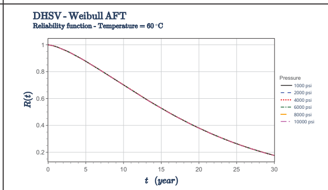
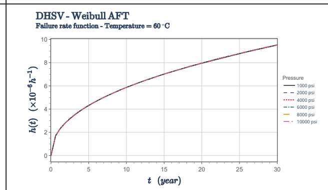
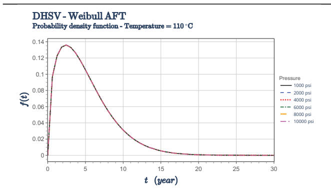
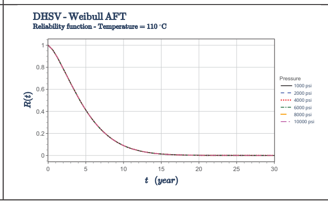
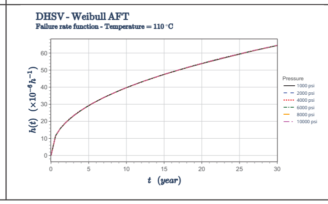
The Cox PH model is semiparametric and uses the Kaplan-Meier estimator. It is a discrete model and therefore reproduces discontinuous reliability curves, i.e., in the form of steps. Probability density functions can only be applied to continuous models; consequently, the failure rate function cannot be plotted.

The Weibull AFT model is parametric and continuous. This means that the curves of the probability density, reliability, and failure rate functions are continuous.

While the Cox PH model seeks the relationship between covariates and the increase or decrease in the failure rate by adopting proportionality, the Weibull AFT model applies covariates to dilate or

Table 8. *

Probability density function, reliability, and failure rate curves for temperature values.

Probability density function	Reliability function	Failure rate function
		
		
		

shorten the effect of operating time.

The Weibull AFT model, when lower temperature values are used, generally provides higher reliability values than those obtained by the Cox PH model.

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References

Akaike, H. (1974). A New Look at the Statistical Model Identification. *IEEE Transactions on Automatic Control AC-19*(6), 716–723.

Cox, D. R. (1972). Regression Models and Life-Tables. *Journal of the Royal Statistical Society: Series B (Methodological) 1*(34), 187–202.

Crowther, M. J., P. Royston, and M. Clements (2023, July). A flexible parametric accelerated failure time model and the extension to time-dependent acceleration factors. *Biostatistics 24*(3), 811–831.

Jerzy K. Filus and Lidia Z. Filus (2017, April). The Cox-Aalen Models as Framework for Construction of Bivariate Probability Distributions, Universal Representation. *Journal of Statistical Science and Application 5*(2), 56–63.

Kaplan, E. L. and P. Meier (1958). Nonparametric Estimation from Incomplete Observations. *Journal of the American Statistical Association 53*(282), 457–481.

Pang, M., R. W. Platt, T. Schuster, and M. Abrahamowicz (2021, January). Spline-based accelerated failure time model. *Statistics in Medicine 40*(2), 481–497.

Wei, L. J. (1992). The accelerated failure time model: A useful alternative to the cox regression model in survival analysis. *Statistics in Medicine 11*(14-15), 1871–1879.