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Limited Memory Influence Diagram Framework for Modeling Condition-Based Maintenance for a System Subjected to Degradation

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Limited memory influence diagrams are useful frameworks for representing and analyzing decision-making under uncertainty and offer the advantage of modeling decision problems with large and complex domains. This paper develops a limited memory influence diagram framework for finding the optimal maintenance strategy for a deteriorating system in the context of condition-based maintenance. The degradation process of the system is modeled by a discretized gamma process. A dynamic Bayesian network framework is applied to describe and update the system state over time based on its degradation process. Inspections are performed at regular intervals corresponding to the time between time slices in the dynamic Bayesian network framework. At each inspection, the system may undertake preventive maintenance activities based on the results from system condition monitoring. The dynamic Bayesian network is extended to a limited memory influence diagram, which additionally includes decision nodes and utility nodes. The decision nodes indicate maintenance actions and the utility nodes indicate the maintenance costs associated with maintenance activities. The decision on repair actions is based on inspection results, and the system is renewed after a given number of inspections. The optimal maintenance strategy is determined so that the expected utility over all possible strategies is maximized. This paper proposes a comparison of optimized maintenance strategies for the system for different inspection time intervals. Numerical examples are given to illustrate the results.

Keywords: Limited Memory Influence Diagram, Dynamic Bayesian Network, Preventive Maintenance, Condition-Based Maintenance, Maintenance Optimization.

1. Introduction

In industrial practice, preventive maintenance (PM) actions play an important role in improving the reliability and availability of maintainable systems. PM is a scheduled maintenance strategy carried out while a system is still functioning. Its goal is to prevent future failures, reduce the failure rate, avoid unnecessary maintenance, and minimize the overall costs related to system failures, downtime, maintenance, and replacements. Depending on the information used for decision-making, maintenance activities are classified into (Rausand and Hoyland (2003)): (i) Time-based maintenance (TBM), where maintenance tasks are carried out at specified calendar times; (ii) Age-based maintenance (ABM), where maintenance tasks are implemented based on a actual system's total operating time; (iii) Condition-based maintenance (CBM), where maintenance tasks are planned based on measurements of one or more variables that indicate the state of the system. These measurements are obtained through system condition monitoring. Maintenance decisions are typically made when the value of a state variable reaches or exceeds a predefined threshold. These thresholds categorize the system's state space into distinct decision areas, each representing a specific maintenance action. Due to its effectiveness and practical importance, CBM has attracted significant attention in the research of maintenance modeling for deteriorating systems (Alaswad and Xiang (2017)). Degradation modeling of a deteriorating system has a profound impact on the CBM decision-making. In recent years, many types of stochastic models have been studied in the literature to describe the degradation processes of industrial systems in the context of CBM. Among these stochastic models, stochastic processes such as the Wiener process (Sun et al. (2017)), the gamma process (Caballé et al. (2015)), and the inverse Gaussian process (Chen et al. (2015)) have been widely used for modeling degradation processes.

In practice, degradation processes are associated with significant uncertainty arising from various random factors such as loading and stress, environmental conditions, and material properties. This uncertainty has been represented by probabilistic models in the past. In recent years, Bayesian methods have emerged as a valuable approach for integrating inspection data into probabilistic deterioration models. These methods effectively quantify the reduction in inherent uncertainty in degradation processes and enable updating the degradation levels of systems and their reliability assessments using Bayes' theorem. This can be achieved using a Bayesian network (BN) framework. BNs are probabilistic graphical models designed to facilitate belief updating under uncertainty (Jensen and Nielsen (2007)). In recent years, BNs have been widely used in many applications for system performance evaluation, risk analysis, and maintenance. Dynamic Bayesian networks (DBNs) are a special type of BNs that can be used to model time-dependent problems, such as stochastic processes. In CBM literature, DBNs have been regarded as effective frameworks for modeling and updating degradation processes of systems modeled by stochastic processes (Straub (2009), Lorenzoni and Kempf (2015)). A DBN-based degradation model can improve the reliability and availability of systems, enable early detection of potential failures, and aid in planning inspections and repair decisions. A DBN framework supports decision-making under uncertainty. A DBN can be extended into an influence diagram (ID) by incorporating decision and utility nodes(Kjaerulff and Madsen (2008)). IDs are effective tools for belief updating and decision-making under uncertainty in a given problem domain. In CBM literature, IDs have been widely studied for maintenance modeling and optimization (Nielsen and Sørensen (2010)). An ID framework can provide an efficient framework for belief updating of the degradation processes of systems and determining optimal maintenance strategies under uncertainty, based on information gathered through condition monitoring. A limited memory influence diagram (LIMID) is a type of ID that explicitly specifies the available information to the decision maker before a decision is made. It explicitly accounts for the possibility that decision makers may not remember past information. LIMIDs can be regarded as effective decision-making tools for representing large and complex domains and multistage decision problems. Due to their graphical structure exploiting conditional independence and limited memory modeling, LIMIDs provide superior computational tractability and interpretability compared to other methods, such as stochastic dynamic programming (SDP) and Markov decision processes (MDP), for maintenance decision-making, especially in large, multistage problem domains. In this paper, we propose a LIMID framework designed to determine the optimal CBM strategy for a deteriorating system. The gradual system degradation phenomenon is modeled using a discretized gamma process. Optimal maintenance decisions are derived from inspection results by maximizing the expected utility over all admissible strategies.

2. Degradation Process of System

We consider a system that starts operating at time 0. The system is subject to deterioration over time, and its degradation level evolution is represented by a continuous-time stochastic process. Let $X(t)$ be the degradation level of the system t time units after its initiation. We assume that $\{X(t); t \geq 0\}$ is modeled by a stationary gamma process with shape parameter α and scale parameter β ($\alpha, \beta > 0$). Therefore, $X(0) = 0$ and the pdf of increment $X(t) - X(s)$, for $s < t$, is given by

$$f_{\alpha(t-s),\beta}\left(x\right) = \frac{x^{\alpha(t-s)-1}\, e^{-\frac{x}{\beta}}}{\beta^{\alpha(t-s)}\Gamma\left(\alpha\left(t-s\right)\right)}, \quad x \geq 0,$$

where Γ denotes the gamma function and is defined as follows:

$$\Gamma(\alpha) = \int_0^\infty u^{\alpha-1} e^{-u}\, du.$$

The system is said failed when its degradation level exceeds the predefined threshold L.

To develop a BN-based tractable modeling framework, the continuous degradation process is discretized in time and space as follows. The discretization time step is denoted by Δt, and the successive discrete times are defined by $t_k = k\Delta t$, $k > 0$. The state space $[0, \infty)$ is partitioned into d bins based on predefined degradation levels $x_0, \ldots, x_{d-1}$ with $x_0 = 0$ and $x_{d-1} = L$. For $1 \leq i \leq d-1$, discrete state number i is $S_i = [x_{i-1}, x_i)$ and $S_d = [x_{d-1}, \infty)$ represents the failure state. Finally, the discretized degradation process is assumed to be a homogeneous Markov chain. It is denoted by $\{Y_k, k \geq 0\}$, where $Y_k = i$ means that the degradation level at time $k\Delta t$ is in state S_i. The transition matrix $H_{\Delta t}$ is empirically estimated from sample paths of the original continuous degradation process. The simplest discretized degradation process is for a system with states $\{S_1, S_2, S_3\}$, where state S_1 denotes the perfect state, state S_2 denotes the degraded state, and state S_3 denotes the failure state. In this case, $d = 3$ with $x_0 = 0$, $x_2 = L$ and x_1 is the threshold between perfect (below x_1) and degraded (above x_1) states.

3. Bayesian Networks

A Bayesian network (BN) is a graphical model that represents probabilistic relationships among a set of random variables. It consists of nodes, each corresponding to a random variable, and directed arcs that depict conditional dependencies between the random variables, forming a directed acyclic graph (DAG). Additionally, each random variable is associated with a conditional probability distribution (CPD) given its parents. In a BN consisting of n nodes $\{A_1, A_2, \ldots, A_n\}$, a directed link from a node A_i to A_j ($i \neq j$) indicates that A_i is a parent of A_j and A_j is a child of A_i. The joint probability distribution over the set $\{A_1, A_2, \ldots, A_n\}$ using the chain rule is defined as follows:

$$P(A_1, A_2, \ldots, A_n) = \prod_{i=1}^{n} P(A_i | pa(A_i)),$$

where $pa(A_i)$ is the set of the parents of A_i. Therefore, a BN encodes a joint probability distribution over a set of random variables in terms of their dependence and independence relationships represented by directed arcs in a DAG. Fig.1 shows a simple BN.

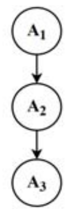

Fig. 1. Example of a BN

A BN enables Bayesian updating of the probabilities through the incorporation of evidence. When new information about one or more random variables is obtained, the joint probability distribution is updated, and posterior probabilities can be found using Bayes' formula.

A dynamic Bayesian network (DBN) extends a BN framework to model the temporal evolution of random variables. A DBN is constructed by replicating a BN across a finite number of time slices. Time slices are connected through temporal links, which allow the incorporation of temporal dependence. A DBN is fully defined when the conditional probability tables for each random variable, conditioned on its parents, are given. Fig. 2 shows an example of a DBN.

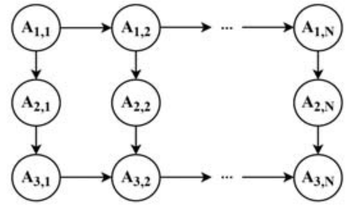

Fig. 2. Example of a DBN

Consider a DBN over N time slices, each consisting of n random variables $\{A_1, A_2, \ldots, A_n\}$. The joint probability distribution over all random variables from time slice 1 to N is defined as follows:

$$P(A_{1:N}) = \prod_{k=1}^{N} \prod_{i=1}^{n} P(A_{i,k} | pa(A_{i,k})),$$

where $pa(A_{i,k})$ denotes the parent of node $A_{i,k}$ for $i = 1, \ldots, n$ and $k = 1, \ldots, N$.

In a DBN, if both the network structure and the conditional probability tables remain the same for all time slices, the DBN is referred to as homogenous. The time slices of a DBN are chosen such that the DBN satisfies the Markov property. Specifically, a DBN is called a first-order Markovian if the variables at time slice $k + 1$ are conditionally independent of those at time slice $k - 1$ given the variables at time slice k. DBNs are particularly effective frameworks for modeling degradation processes, and the process is often simplified by assuming it adheres to the first-order Markov property.

3.1. *DBN Framework for Condition Modeling of Deteriorating System*

Figure 3 shows a DBN framework developed to describe the evolution of the maintained system state based on its discretized degradation process, $\{Y_k, k \geq 0\}$, for N time slices, which corresponds to the number of inspections over the total system replacement cycle. The node D_k represents the system state at time slice $k \in \{1, \ldots, N\}$, with state space dom(D_k)= $(S_1, \ldots, S_d)$, $d > 1$, which S_1denotes the perfect state and S_d denotes the failure state. The node R_k represents the inspection result at time slice $k \in \{1, \ldots, N\}$, with state space dom(R_k)= $(I_1, \ldots, I_d)$, $d > 1$. Inspections are performed every ΔT time unit, corresponding to the time between successive time slices in the DBN. In accordance with the discretized degradation process Y_k described in section 2, ΔT is equal to $\eta \Delta t$ where η is the number of discrete time steps between two successive inspections. The transition probability matrix of the system state between two successive inspections is then $H_{\Delta T} = H_{\Delta t}^{\eta}$. We assume inspection times are negligible, and inspections are perfect in the sense that the degradation of the system is observed with certainty. Therefore, $I_k = S_k,\ k = 1, \ldots, N$.

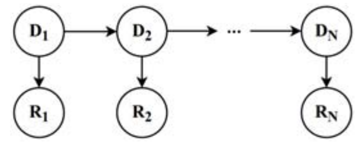

Fig. 3. A DBN for system condition modeling

4. Influence Diagrams

An Influence diagram (ID) is a framework for representing and solving decision problems under uncertainty. An ID is a DAG that consists of three types of nodes: chance nodes (circles), which represent random variables; decision nodes (rectangles), which represent decision variables; and value nodes (diamonds), each of represent a utility function. Each utility function assigns a utility value to each possible configuration of its parent nodes. The directed links encode the dependence relations and information precedence. The structure of an ID is based on two fundamental assumptions: (i) a total order on decisions, which ensures that the ID defines a temporal sequence of decisions, and (ii) the "no-forgetting" assumption, meaning that when making a decision, the decision maker remembers all past observations and decisions. An ID can be used to compute expected utility for the various decision alternatives. Solving an ID amounts to determining the optimal strategy and computing its expected utility. A strategy for an ID is a set of decision policies, one for each decision variable. In this framework, the optimal strategy is the one that maximizes the expected utility by selecting the optimal set of policies.

The assumptions underlying IDs can lead to considerable computational complexity, particularly for large and complex domains and multistage decision problems. To address this challenge, limited memory influence diagrams (LIMIDs) are proposed. A LIMID relaxes fundamental assumptions of ordinary ID representation by explicitly identifying the information that is available to the decision maker when each decision is made. This refinement significantly reduces the size of the policy domain and facilitates the strategy optimization.

4.1. *LIMID for Maintenance Decision Making*

Considering the system under study, Fig. 4 shows a LIMID framework for maintenance decision making over N time slices. The memory for decision-makers is bounded to the current time slice. The framework is constructed based on the DBN framework shown in Fig.3 augmented with decision nodes, M_k, utility nodes $U_{M,k}$, $k = 1, \ldots, N$, and random variables D'_k, $k = 1, \ldots, N-1$, with state space dom(D'_k)= $(S_1, \ldots, S_d)$, $d > 1$.

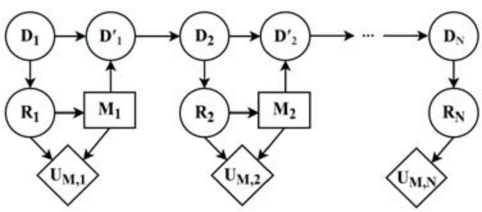

Fig. 4. A LIMID for maintenance modeling

The decision node M_k represents the maintenance decision options at time slice k. The Maintenance decisions are made at inspection times based on the inspection results. At each inspection, we have two decision options: do nothing and repair. Repairs are assumed to be perfect, meaning that after a preventive or corrective repair, the system returns to as-good-as-new state.

The utility node $U_{M,k}$ captures both the inspection cost and the cost of any maintenance action decided to be carried out at time $k\Delta T$. The node D'_k denotes the state of the system at time $k\Delta T$ immediately after the maintenance action specified in M_k. The system is fully replaced with a new one at N-th inspection or equivalently at time $T = N\Delta T$. The associated replacement cost, C_r, is represented by the utility $U_{M,N}$. The inspection cost is C_i, the preventive repair cost is C_p, the corrective repair cost is C_c. In case of failure, an additional cost C_f is considered, whatever the decision about maintenance is. The optimal maintenance strategy is the one that maximizes the expected utility over the time between two system replacements.

5. Numerical Example

The numerical experiments have been obtained using the Julia language and three main packages that are `DecisionProgramming` for problem formulation with IDs, `JuMP` for modeling interface, and `Gurobi` as an optimization solver.

As an example, we consider a deteriorating system whose continuous degradation process is modeled by a homogeneous gamma process with a shape parameter $\alpha = 2$ and scale parameter $\beta = 1$. The mean degradation rate is then equal to 2 per time unit. The LIMID proposed in subsection 4.1 is implemented for condition-based maintenance optimization with different inspection intervals (ΔT) and inspection numbers (N). The utility nodes are specified based on the maintenance costs $C_i = 1$, $C_p = 7$, $C_c = 15$, $C_f = 24$, and $C_r = 164$. The expected utilities are negative, indicating that the utility nodes are defined in terms of maintenance costs. To facilitate interpretation, these values will be treated as positive throughout this section.

5.1. *Case of a Short Replacement Duration*

We implemented the proposed LIMID for a system with $d = 3$ condition states over $N = 5$ time slices. We set $L = 8$, and the system is fully replaced with a new one at the 5th inspection performed and inspections are performed every $\Delta T = 1$ unit. Transition probabilities between states are estimated by a Monte Carlo simulation of 9000000 degradation paths, and the time step for gamma process discretization is assumed to be $\Delta t = 1$. Table 1 shows the optimal maintenance strategies based on observation obtained from inspection results for time slices $k = 1, \ldots, 4$. For each time slice, the optimal maintenance actions are presented based on inspection outcomes. These strategies are encoded as binary vectors, where 1 indicates “do nothing” and 0 indicates "repair". The expected utility is $EU = 178.96$. The decision maker takes no action when the inspection indicates that the system is in the perfect state S_1, carries out repairs when the inspection results are I_2 or I_3, and takes no action at the last inspection, regardless of the inspection outcome.

Table 1. Optimal maintenance decision strategies for $T = 5$ and $L = 8$

Time Slice	Maintenance Strategies
1	[1, 0, 0]
2	[1, 0, 0]
3	[1, 0, 0]
4	[1, 1, 1]

Fig. 5 illustrates the total expected utilities in terms of various values for N and ΔT. This graph is generated based on the proposed LIMID for ΔT, ranging from 1 to 5, and over time slices, N, changing from 2 to 10. Inspections are assumed to be carried out every $\Delta T = 1$ unit. The time step for the degradation process discretization is set at $\Delta t = 1$ for each configuration. Based on the Figure 5, the expected utility increases in ΔT when N is fixed, that is, for a fixed number of time slices (fixed number of inspections), the expected utility increases as the time between inspections decreases. This happens with significant intensity for high values of N. Therefore, to attain a higher expected utility—especially with a greater number of time slices and longer replacement durations—it is advisable to set the time between inspections to smaller increments.

In another configuration, the system replacement duration is of the same order of magnitude as the system's lifetime. Let consider $T = 5$. Figure 6 illustrates the evolution of the optimal decision map for different configurations of gamma process discretization and different times between inspections. The time step for gamma process discretization is $\Delta t = 0.2$. The dark area (blue) represents the decision not to perform maintenance, and the light area (yellow) corresponds to a decision to perform maintenance. For one inspection at each 0.2 time unit ($\Delta T = 0.2$), the

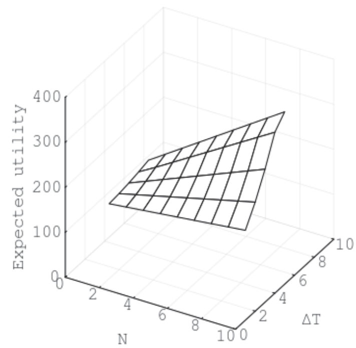

Fig. 5. Expected utility in terms of N and ΔT

expected utility is equal to 195.1 if $d = 5$ and 194.03 if $d = 41$. For one inspection at each time unit ($\Delta T = 1$), the result is respectively 175.97 for $d = 5$ and 173.32 for $d = 41$. As expected, a finer discretization of degradation level allows for a lower optimal cost. However, it should be noted that the difference is small. In that case, an inspection every time units is economically preferable.

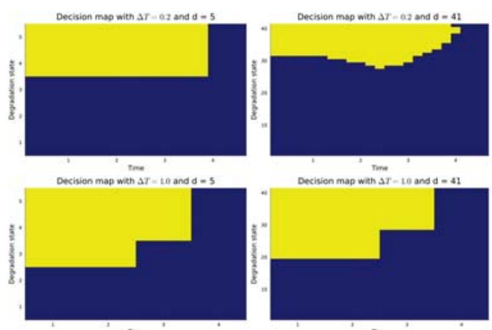

Fig. 6. Decision maps for $T = 5$ with $\Delta t = 0.2$ and different values of ΔT and d.

Results given in Figure 7 focus again on the configuration with $\Delta T = 1$ but have been obtained with a larger discretization time step $\Delta t = 1$ for the gamma process. The expected utility is equal to 176.8 for $d = 5$ and 176.04 for $d = 41$, which is slightly different from that with $\Delta t = 0.2$. It shows the influence of the discretization

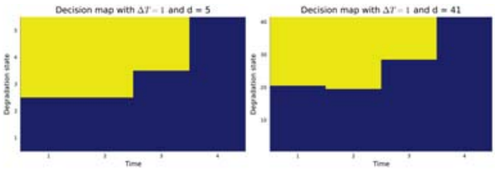

Fig. 7. Decision maps for $T = 5$ with $\Delta t = 1, \Delta T = 1$ and two different values of d.

stage on the obtained optimal result.

Table 2 shows the expected utilities obtained from solving the proposed LIMID for condition-based maintenance strategy optimization in terms of different numbers for system states (d) and for the failure threshold (L). We assume $N = 5$ and $\Delta T = 1$. Moreover, d changes from 3 to 8 states, and L is set at levels 8, 12, and 16. The degradation process discretization is obtained for time step $\Delta t = 1$. Obviously, the value of expected utility tends to decrease as the failure threshold L increases, i.e. as the system lifetime increases. For each value of failure threshold, the expected utility decreases as d increases. This result indicates that independently of the value of L, a lower expected utility is achieved when the system's degradation level is categorized into more states for maintenance decision-making.

Table 2. Expected utilities

d		L	
	8	12	16
3	178.96	174.4	173.35
4	178.388	174.244	173.134
5	176.88	173.335	171.803
6	176.658	172.227	171.012
7	176.284	171.47	169.874
8	176.141	171.14	169.525

Table 3 gives the decision strategy obtained for $d = 8$ and $L = 16$ as a reference for section 5.3.

5.2. *Case of a Long System Replacement Duration*

This configuration considers the case where the replacement duration time is significantly longer than the system's lifetime. Then several repairs

Table 3. Decision strategies

Time slice	Decision Strategies
1	[1, 1, 1, 1, 0, 0, 0, 0]
2	[1, 1, 1, 1, 1, 0, 0, 0]
3	[1, 1, 1, 1, 1, 1, 0, 0]
4	[1, 1, 1, 1, 1, 1, 1, 1]

are necessary before replacement. Figure 8 illustrates the evolution of the optimal decision map for $T = 50$ with one inspection per time unit ($\Delta T = 1$) and for different refinements in the discretization of the degradation space, respectively $d = 5, 11, 41$, and 81. The respective expected utilities are 365.79, 364.71, 365.10, and 364.22.

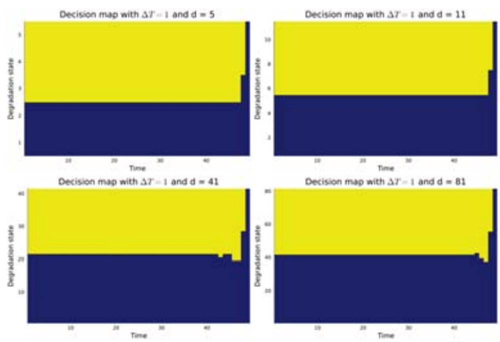

Fig. 8. Decision maps for $T = 50$ with $\Delta t = 1$ and $\Delta T = 1$ for different values of d.

In all cases, the maintenance decision can be simplified to selecting a constant threshold between "do nothing" and "repair" for almost the entire replacement duration. Toward the end of the mission (from around time 45), this threshold adjusts according to the degradation level in order to minimize the expected utility.

5.3. *CBM Strategy Optimization Under Imperfect Inspection*

Here we consider the situation in which the inspections are not perfect, so I_k is no longer equal to S_k. Imperfect inspection is less expensive than perfect inspection, but it may yield false positives or false negatives in failure detection. We implemented the proposed LIMID for a system under imperfect inspection over $N = 5$ time slices when $\Delta T = 1$. We assume that $d = 8$, $\Delta t = 1$ and $L = 16$. At each inspection, the exact state of the system is accurately observed with probability 0.95 and its state is incorrectly observed as failed with probability 0.05. The failure of the system is accurately detected with probability 0.86, and is incorrectly observed at state S_7 with probability 0.14. We assume the inspection cost is $C_i = 0.5$. The Table 4 shows the optimal maintenance strategies obtained for the system under imperfect inspection. The expected utility is $EU = 171.768$, which does not have a considerable difference from the result for expected utility $EU = 169.525$ obtained for the system with the same number of states and failure threshold in subsection 5.1. Compared with Table 3, we see that the decision maker decides to do nothing when the system fails. Moreover, the number of repairs decreases when inspections are imperfect.

Table 4. Decision strategies

Time slice	Decision Strategies
1	[1, 1, 1, 1, 0, 0, 0, 1]
2	[1, 1, 1, 1, 1, 0, 0, 1]
3	[1, 1, 1, 1, 1, 1, 0, 1]
4	[1, 1, 1, 1, 1, 1, 1, 1]

6. Conclusion

This paper presented a LIMID framework for condition-based maintenance optimization for a system subject to deterioration. The optimized maintenance strategies are identified so that the expected utility associated with maintenance costs is maximized. Numerical examples illustrate the influence of the time between inspections and the number of repairs over a replacement cycle on maintenance decision-making. The results show that the failure thresholds and number of states defined for the system have a significant influence on the number of repairs over a replacement cycle, thereby achieving higher expected utility. Furthermore, the proposed framework is implemented for the case where the inspections carried out during a replacement cycle are imperfect.

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