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Interval Uncertainty Propagation on CO_2 Emission Calculations in Road Haulage

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ISO 14083:2023 was introduced to guide the calculation of greenhouse gas emissions within the logistics industry. Under this standard, the uncertainty of reported emissions values can, at best, be inferred from whether the value is based on measurements, generated by modeling or using industry default values. This makes it difficult to gauge the uncertainty of emissions figures used in decision-making. This work aims for transparency through explicit quantification of uncertainty. Large corporations are required to report on their emissions, including those made on their behalf. However, road haulage is dominated by small companies that are exempt from emissions reporting, and many do not provide their customers with emissions data.

This paper considers the position of small road haulage companies that can take measurements, as opposed to companies that subcontract logistic operations. For simplicity, the paper only considers direct CO_2 emissions. This work provides an analysis of the uncertainty that would arise from collecting data on fuel usage, cargo weights, and distances, and characterises it using intervals. An exemplar road haulage scenario is then described. The proposed method has low computational overheads and generates bounds that are not inflated by the propagation mechanism when applied in the context of ISO 14083:2023. Since this methodology is applicable across the logistic industry, similar analysis performed in a different part of the industry should reach comparable conclusions.

Keywords: Interval arithmetic, Uncertainty quantification, Greenhouse gas emissions, Logistics, Road haulage.

1. Introduction

When a parcel arrives at a door, it has often been on a long journey. It may have crossed oceans in container ships, flown through the skies in an aeroplane, traveled hundreds of kilometers in the back of lorries and trucks, visited numerous warehouses and been taken on a tour of the local neighbourhood in a van. Each step was shared with countless other items each with their own destinations and paths. This activity of moving goods to where they are needed in an efficient manner is the work of the logistics industry.

The ships, planes, trucks, lorries and vans that keep the logistic industry moving are a major source of greenhouse gases. For decision makers from companies choosing which logistic contracts to sign, to a customer choosing what goods they will buy and how to receive them, it is key to be informed about both the amount and the uncertainty of emissions. Calculating emissions for individual parcels, pallets and containers requires an account of the overall emissions for a vehicle or vehicles over a period of time, dividing those emissions between all the goods that have been moved, and a way to sum up all the emissions for a journey through the entire logistics system.

The contribution of this paper is the application of interval arithmetic to create a methodology for uncertainty quantification of emissions allocation within logistics. It is based on ISO 14083:2023, a standard for calculating emissions in the logistics sector that allocates emissions between users of a logistic service. The intended users of the method set out in this paper are smaller logistics companies that do not have the resources to use more established methods of uncertainty quantification such as Monte Carlo methods.

The road haulage industry is dominated by small businesses. In the UK, 92.1% of road haulage companies have between zero and nine employees and only one company has a market share over six percent Atkins Realis (2024). Each leg of an item's journey will often have an external logistic service provider that frequently subcontract to smaller operators, creating a complex system of organisations for emissions data to pass through. Many of the small companies that are the ultimate source of the data do not believe they have the capability to provide emissions estimates Toelke and McKinnon (2021). Regardless of the structures of the organisation, there are a few common patterns for road delivery, with two common variants are outlined below.

The most basic pattern for a logistic operation is a point to point model where a vehicle starts at a location A loads on some goods to be delivered, moves to the destination B, unloads the goods onboard and returns empty back to A.

This model is sometimes used for the delivery of goods between factories and distribution centers, or in other scenarios where reliable transportation is business critical. Since the vehicle is travelling empty half the time, other patterns that can achieve higher capacity utilisation are often used. Another pattern, involves a vehicle starting at a warehouse or logistics hub, loading goods for multiple locations onboard, then delivering to each destination before returning to the beginning. In some cases, goods to be moved to the warehouse are loaded onto the vehicle near the end of the journey, but this is often considerably less than the amount of goods leaving.

2. ISO 14083:2023

ISO 14083:2023 International Organization for Standardization (2023) sets out a standardised way to calculate and distribute the emissions related to logistical activities between service users and individual items. It sets out key concepts and terms which we describe in what follows.

Emissive activities are those that create emissions. This paper will only consider the combustion of diesel fuel, but electricity consumption could also be considered.

Transport activities concern the transport of items from one place to another. They have start and end locations and an amount to be moved. Those amounts are often measured in tonnes or kilograms, but, in some cases, other measurements can also be used.

Transport activity categories are sets of transport activities that have similar characteristics, for example those done by the same or similar vehicles. This paper assumes that the transport activities of a vehicle all fall into the same transport activity categories.

Round trips are required by ISO 14083:2023. In particular, it requires that emission calculations for transport activities by moving vehicles are based on round trips. Vehicles must return back to their original starting position, or if that is not feasible in practice, then the transport activity category should be over such a sufficiently varied set of transport activities that the direction of movement is not likely to change the emission amount.

Shortest feasible distance (SFD) is the shortest distance to travel along a transport network to move from a starting to an end location. For a lorry this would be the shortest route along roads that do not have height or weight restrictions.

Emission factors are values representing the emission amount for each unit of emissive activity. $3.16433\ \mathrm{kg\,CO_2\,kg^{-1}}$ is an emission factor for diesel fuel combustion provided by the UK Government Department for Energy Security and Net Zero (2025). ISO 14083:2023 provides its own set of emission factors but encourages the use of more localised factors where they are available and reliably produced.

For a transport activity category where emissive activity data is available, ISO 14083:2023 sets out formulae to calculate the emissions per item. Total emissions of a transport activity category should, where possible, be calculated by multiplying the amount of all emissive activities by appropriate emission factors. The total emissions should then be shared between transport activities by the relative amount of these activities.

The model of multiplying emissions factors by activity data is a simplification and the ISO 14083:2023 allows the use of oher models but the authors have chosen to focus on this model as it is the one explicitly written in the standard.

Definition 2.1 (ISO Formula). *Let us consider a small road haulage company with all the information about the transport activities, item weights, distances and fuel consumption available for a given transport activity category. The emissions attributable to the movement of a given item β in a given transport activity α is given by*

$$e_{\alpha\beta} = \frac{m_{\alpha\beta} d_\alpha \sum_i f_i c_i}{\sum_j \sum_{k(j)} m_{jk} d_j}. \tag{1}$$

where:

- i *index enumerating the n emission activities of a transport activity category, $i = 1, ..., n$.*
- j *index enumerating the p transport activities of a transport category, $j = 1, ..., p$.*
- $k(j)$ *index enumerating the $l(j)$ items being moved in the activity j, $k(j) = 1, ..., l(j)$.*
- α *transport activity of interest, α takes one of the values of j.*
- β *item of interest, β takes one of the values of $k(\alpha)$.*
- $e_{\alpha\beta}$ *emissions attributable to the movement of item β in the transport activity α, measured in* $\mathrm{kg\,CO_2}$.
- m_{jk} *weight of the item k for the activity j, measured in* t.
- d_j *shortest feasible distance for the activity j, measured in* km.
- f_i *emission factor for the emission activity i, measured in* $\mathrm{kg\,CO_2\,kg^{-1}}$.
- c_i *amount of emission activity i, e.g. fuel consumption, measured in* $\mathrm{kg\,CO_2}$.

3. Uncertainty Methodologies

Research on propagating emission uncertainties is focussed on national emission inventories. (Frey et al., 2006, Chapter 3, pg 27) calls for the use of Monte Carlo or error propagation formula. Since then attempts to refine the Monte Carlo method include an effort at controlling the samples Dey et al. (2019). Eltohamy et al. (2024) studied lifecycle analysis of electric vehicles and found that uncertainty analysis is rare and uses Monte Carlo methods when performed.

These methods can be unsuitable for some logistics operators. As stated above, many small logistics operators that will struggle to perform analysis to find probability distributions of their input data. Of course an assumption about the distribution can still be made, but this would be a strong assumption that could lead to false confidence Carmichael and Williams (2018).

Another approach to uncertainty is to use interval arithmetic as set out by Moore et al. (2009) to get upper and lower bounds on a value. Quantifying the uncertainty with intervals is more accessible and have fewer strong assumptions. Intervals can be propagated through existing formula only requiring knowledge of the endpoints of the parameters.

3.1. *Interval arithmetic*

Interval arithmetic is a way to propagate uncertainty through a series of operations. For intervals that only contain positive real numbers, the common arithmetic operations of addition, multiplication and division respectively simplifies to

$$[a, b] + [c, d] = [a + c, b + d], \tag{2}$$

$$[a, b] \cdot [c, d] = [a \cdot c, b \cdot d] \tag{3}$$

where $a, b, c, d \in \mathbb{R}_0^+$, and

$$\frac{[a, b]}{[c, d]} = \left[\frac{a}{d}, \frac{b}{c}\right] \tag{4}$$

where $a, b \in \mathbb{R}_0^+$ and $c, d \in \mathbb{R}^+$.

3.2. *Interaction between variables*

Any operation combining uncertain numbers must consider how these numbers interact. For two uncertain numbers there are infinitely many ways they could interact. Consider the diesel consumption measured in kg of a vehicle as it travels a distance given in km. Intuitively the values of these numbers should be related to each other, it is expected that driving further would require more fuel, however there are many unknowns that can confound that dependence like the weight of goods on board. By gathering data the nature of the interaction or dependence between variables can become clearer and allow a more complex handling of dependences. It is common in road haulage that such data is unavailable and only two special cases of dependence should be considered: The way the variables interact is unknown or that the variables are the same.

Unknown Dependence One may wish to make no assumptions about how uncertain numbers interact. A change in the value of a number could have a dramatic change in the probability distribution, a minor change or none at all in the other numbers. Without knowledge of how numbers interact these must be treated as non dependent Ferson et al. (2004). Intervals provide a natural framework to compute with unknown dependencies as their baseline operational condition.

Repeated variables Interval arithmetic produces the tightest possible bounds when the intervals are non interactive Ferson et al. (2004). However, when there is knowledge available that allows a stronger dependence assumption, it will produce bounds that are wider than they ought to be. Intervals that are perfectly correlated or repeated in an expression are such a case. In Equation (1), the variables d_α and $m_{\alpha\beta}$ are repeated in the numerator and denominator when $j = \alpha$ and $k = \beta$.

Theorem 1. *Let g be a componentwise monotone function with respect to all of its parameters. Then:*

(a) The maximum of g can be found using the left endpoint of all non-increasing parameters and the right endpoint of all non-decreasing parameters.

(b) The minimum of g can be found using the right endpoint of all non-increasing parameters and the left endpoint of all non-decreasing parameters.

Proof. A proof of this result can be found in (Moore et al., 2009, Theorem 6.3, pg 73). □

Proposition 1. *Let us consider the emissions attributable to the movement of item β in the transport activity α described in Definition 2.1. Then, $e_{\alpha\beta}$ is non decreasing with respect to c_i, and f_i, for all $i = 1, \ldots, n$, and, d_α and $m_{\alpha\beta}$. Moreover, it is non increasing with respect to d_j for all $j \neq \alpha$ and with respect to m_{jk}, for all $j = 1, \ldots, m$ and all $k = 1, \ldots, l(j)$ except $m_{\alpha\beta}$.*

Proof. The proof of this proposition can be obtained by partially differentiating Equation (1) by each parameter and noticing that the corresponding derivatives are non negative and non positive within their respective intervals. □

By combining Theorem 1 and Proposition 1 we have that:

(a) The maximum of $e_{\alpha\beta}$ can be found using the lower bounds of the intervals associated to the quantities d_j for all $j \neq \alpha$ and m_{jk} for all $j = 1, \ldots, m$ and all $k = 1, \ldots, l(j)$ except $m_{\alpha\beta}$; and the upper bounds of the intervals associated to the quantities c_i, f_i, for all i, d_α and $m_{\alpha\beta}$.

(b) The minimum of $e_{\alpha\beta}$ can be found using the upper bounds of the intervals associated to the quantities d_j for all $j \neq \alpha$ and m_{jk} for all $j = 1, \ldots, m$ and all $k = 1, \ldots, l(j)$ except $m_{\alpha\beta}$; and the lower bounds of the intervals associated to the quantities c_i, f_i, for all i, d_α and $m_{\alpha\beta}$.

It follows that the bounds on emission activities can be computed exactly as shown in the two formulas below where underlines represent the minimum and overlines represent the maximum, respectively.

$$\overline{e_{\alpha\beta}} = \frac{\overline{m}_{\alpha\beta}\overline{d}_\alpha \sum_i \overline{f}_i \overline{c}_i}{\overline{m}_{\alpha\beta}\overline{d}_\alpha + \overline{d}_\alpha \sum_{k(\alpha)\neq\beta} \underline{m}_{\alpha k} + \sum_{j\neq\alpha}\sum_{k(j)} \underline{m}_{jk}\underline{d}_j}, \tag{5}$$

$$\underline{e_{\alpha\beta}} = \frac{\underline{m}_{\alpha\beta}\underline{d}_\alpha \sum_i \underline{f}_i \underline{c}_i}{\underline{m}_{\alpha\beta}\underline{d}_\alpha + \underline{d}_\alpha \sum_{k(\alpha)\neq\beta} \overline{m}_{\alpha k} + \sum_{j\neq\alpha}\sum_{k(j)} \overline{m}_{jk}\overline{d}_j}. \tag{6}$$

This result does not rely on independence assumptions and gives bounds that are the tightest possible that will hold regardless of the distrubutions or interactions between variables.

4. Example Usage

We present a two-part purely illustrative example scenario chosen to facilitate the understanding of the calculation process. The examples show one way to apply the ISO 14083:2023 both with and without quantified uncertainties. The underlying data used in the examples is fictitious, but the distances and fuel consumption values were derived using existing tools from the underlying data.

A fulfillment center for a manufacturing company receives goods from a nearby factory picked up and delivered everyday by their own large goods vehicle. It also receives orders for those goods from a number of retailers around the country, it sorts and picks out those orders and arranges for their delivery with logistics operators. One of the retailers has asked for emissions statements for each of their orders. The retailer has requested an estimate of the CO_2 emissions arising from delivery of four orders of $1\,\mathrm{tonne}$, $2\,\mathrm{tonne}$, $2\,\mathrm{tonne}$ and $3\,\mathrm{tonne}$ they have received.

The distances used were generated by using open source mapping software and map data to create routes between locations representative of road freight activity in the UK OpenStreetMap (2025); Openrouteservice (2025). The great circle and shortest feasible distances between the factory and fulfillment center are $22.2\,\mathrm{km}$ and $24.5\,\mathrm{km}$ respectively. There are five locations including the fulfillment center used in the second part of the example.

4.1. *Variable uncertainty characterisation*

In the examples the intervals of variables are mostly based on governmental regulation and resolution and accuracy of measuring devices.

Weight of goods. As measured values, the uncertainty of m_{jk} and $m_{\alpha\beta}$ in Equation (1) is dependent on the measurement method used. The example assumes class 4 pallet scales with a $10\,\mathrm{kg}$ scale interval should have a maximum possible error of $\pm 1\%$ up to $500\,\mathrm{kg}$, $\pm 2\%$ up to $2000\,\mathrm{kg}$ and $\pm 3\%$ up to $10\,000\,\mathrm{kg}$ if devices covered by EU and UK regulations United Kingdom Parliament (2016b); European Commission (2014) are used.

Distances. The distances d_i and d_α in Equation (1) are SFDs. Ways to bound the distances used here are: (i) A direct path between the two locations as if traveling along a circle the same diameter as the Earth, known as the Great Circle Distance would provide a lower bound. (ii) The distance actually traveled by a vehicle when moving between the points could be an upper bound. However, since vehicles may not be directly traveling between the two points, possibly visiting multiple other locations in between, it may be particularly high. Another issue with using the actual traveled distance as an upper bound is that it conflicts with ISO 14083:2023 that requires an adjustment factor to be used to increase any actual traveled distances used. For the examples, the great circle distance is used as a lower bound and an assumption of the shortest feasible distance $+10\%$ is made for the upper bound.

Fuel consumption. The value of c_i in Equation (1) is a measured value. The examples assume that fuel consumption is measured by fuel receipts created when purchasing fuel. UK legislation requires that measurements of fuel under $100\,\mathrm{L}$ used for transaction have a maximum possible error of $+2\%/-1\%$ United Kingdom Parliament (2016a). For fuel consumption measured in L it is sometimes necessary to convert into kg using fuel density. EN 590 regulates the density of diesel in most of Europe to $[0.820\,\mathrm{kg\,L^{-1}}, 0.845\,\mathrm{kg\,L^{-1}}]$ European Committee for Standardization (2025). Interval arithmetic multiplication Equation (3) between volume and density achieves the conversion.

Emission factors. In this work, we use the emission factor $3.164\,33\,\mathrm{kg\,CO_2\,kg^{-1}}$ is used for all emission calculations in the example and comes from the UK government's emission factors for reporting Department for Energy Security and Net Zero (2025). It only gives a value for the CO_2 released and no other greenhouse gases and assumes that the diesel fuel is pure mineral diesel that does not contain additives or impurities and that it is fully combusted. Since the emission factor represents a simplified model of emissions, there is a large variation in uncertainty involved in emission factors, which we aim to address in subsequent work. For this paper we assume there is no uncertainty in the emission factor used.

4.2. *Scenario 1 – From the factory*

The first part of the scenario is a recurring trip between a factory and the fulfillment center. Fuel receipts for the vehicle driving this route are collected each month. For the month in question, $1476\,\mathrm{L}$ of diesel fuel at $15\,°\mathrm{C}$ was purchased for the $100\,\mathrm{L}$ fuel tank vehicle. Over that month, a total of $1540\,\mathrm{t}$ of goods were moved from the factory to the fulfillment center by the vehicle.

The fuel consumption in kg is given by $1476\,\mathrm{L} \cdot 0.845\,\mathrm{kg\,L^{-1}} = 1247\,\mathrm{kg}$ when calculated without uncertainty. When uncertainty is considered: The lower limit on fuel consumption is given by multiplying the measured fuel consumption by 0.99 for the -1% measurement accuracy minus the $100\,\mathrm{L}$ of fuel that may remain in the tank. The upper limit is given by the measured fuel consumption by 1.02 for the $+2\%$ measurement accuracy plus the $100\,\mathrm{L}$ of fuel that may have been in the tank initially. The maximum and minimum can be combined into an interval for the fuel consumption in L. As previously stated that interval can be multiplied with the regulatory fuel density of $[0.820\,\mathrm{kg\,L^{-1}}, 0.845\,\mathrm{kg\,L^{-1}}]$ to get an interval for the fuel consumption of $[1116\,\mathrm{kg}, 1345\,\mathrm{kg}]$. Note that most of this uncertainty comes from the fact that there are unknown quantities of fuel in the tank at the beginning and end of the time period. A policy change to refill the tank at the end of each month or to move the cutoff points of each transport activity category to just after the last refuel in a month would reduce the uncertainty here dramatically. Even assuming the tank is at least 80% full at the beginning and end would tighten the bounds in this case to $[1181\,\mathrm{kg}, 1290\,\mathrm{kg}]$.

The distance is the shortest feasible distance between the factory and fulfillment center of $24.5\,\mathrm{km}$, this is bounded from below by the great circle distance $22.2\,\mathrm{km}$ and above by an estimated margin of 10% i.e. $24.5\,\mathrm{km} \cdot 1.1 \approx 27.0$ giving an interval of $[22.2\,\mathrm{km}, 27.0\,\mathrm{km}]$.

All of the goods were transported on pallets that were weighed before transportation to the factory, the sum of which was $1540\,\mathrm{t}$. Assuming in the worst case each pallet weighed between $2\,\mathrm{t}$ and $10\,\mathrm{t}$ the measurements would be accurate within $\pm\ 3\%$ giving an interval for the total weight of $[1493\,\mathrm{t}, 1586\,\mathrm{t}]$. In practice it can sometimes be seen as impractical to weigh every pallet transported and options to minimise required measurements is covered in Section 5.

Applying Equation (1) to this can be simplified by noting that d_j is the same for all values of j in this case. When uncertainty is not taken into account the values of d_α and d_j cancel out, and thus for the $1\,\mathrm{t}$ shipment this gives the value of

$$\frac{1\,\mathrm{t} \cdot 1247\,\mathrm{kg} \cdot 3.164\,33\,\mathrm{kg\,CO_2\,kg^{-1}}}{1540\,\mathrm{t}}.$$

However, when considering uncertainty, the division does not cancel out, as is demonstrated in Section 3.2, but d_j can still be pulled outside of the summation $\sum_j \sum_{k(j)} m_{jk} d_j = d \sum_j \sum_{k(j)} m_{jk}$ where $d = d_j, \forall j$.

While using interval arithmetic, Equation (1) works in a similar way. Whilst the items of interest to the retailer were transported as part of the $[1493\,\mathrm{t}, 1586\,\mathrm{t}]$ transported from the factory, the uncertainty of their measurements are not, so the cancellation of repeated variables in 3.2 does not apply in this case and Equation (1) can be used unchanged, albeit with interval arithmetic. The emissions for each of the retailers orders are summarised in Table 1.

Table 1.: The emissions for each of the retailers four orders in scenario 1 presented with and without uncertainty intervals.

Weight (t)	Emissions ($\mathrm{kg\,CO_2}$)	Interval Emissions ($\mathrm{kg\,CO_2}$)
1.0	2.56	[2.18,2.91]
2.0	5.12	[4.31,5.88]
2.0	5.12	[4.31,5.88]
3.0	7.68	[6.47,8.81]

4.3. *Scenario 2 - To retailers*

The second scenario involves a vehicle starting at the fulfillment center (point *A*) with a full tank of diesel and loaded with a number of pallets to deliver to points *B,C,D* and the inquiring retailer at point E, that it visits in order and unloads any goods for that location before continuing to the next point eventually returning to A and refueling back to full. When it visits B, it picks up a pallet destined for C, the requirement to move goods neither from or to the warehouse at A is unusual and has been added to show that calculation method can support it rather than to be representative of standard practices. At E a number of empty pallets are loaded into the vehicle to be returned. The fuel used to refuel was $65.196\,\mathrm{kg}$. The path taken by the vehicle and the total movement of goods between points can be seen in Figure 1. When the same data is used but using interval arithmetic, the results are shown in Table 3. The total CO_2 emissions for the journey are $206.302\,\mathrm{kg\,CO_2}$ obtained from $65.196\,\mathrm{kg} \cdot 3.164\,33\,\mathrm{kg\,CO_2\,kg^{-1}}$. As an interval the total emissions are $[204.238\,\mathrm{kg\,CO_2}, 210.428\,\mathrm{kg\,CO_2}] = [64.544\,\mathrm{kg}, 66.500\,\mathrm{kg}] \cdot 3.164\,33\,\mathrm{kg\,CO_2\,kg^{-1}}$. The different logistic activities involved and the way the emissions are shared between them can be seen in Table 2.

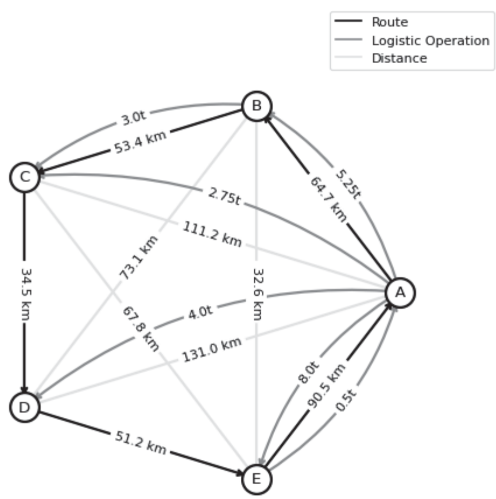

Fig. 1.: The path for the scenario 2. Distances shown are shortest feasible distances. When a distance coincides with the route taken, the line is darker and arrowheads show the direction of travel.

Table 2.: The logistic activities undertaken in scenario 2 along with the emissions when calculated without uncertainty.

Start	End	Weight (t)	SFD (km)	Emissions (kg CO_2)
A	B	2.5	64.7	15.9
A	B	2.8	64.7	17.5
A	C	2.8	111.2	30.1
A	D	1.5	131.0	19.3
A	D	2.5	131.0	32.2
A	E	1.0	90.5	8.9
A	E	2.0	90.5	17.8
A	E	2.0	90.5	17.8
A	E	3.0	90.5	26.7
B	C	3.0	53.4	15.8
E	A	0.5	90.5	4.5

Table 3.: The logistic activities undertaken in the scenario along with the emissions when calculated with uncertainty.

Start	End	Weight (t)	SFD (km)	Emissions (kg CO_2)
A	B	[2.4,2.6]	[58.2,71.2]	[12.1,20.9]
A	B	[2.6,2.9]	[58.2,71.2]	[13.3,23.2]
A	C	[2.6,2.9]	[100.1,122.3]	[22.7,39.6]
A	D	[1.4,1.6]	[117.9,144.1]	[14.6,25.3]
A	D	[2.4,2.6]	[117.9,144.1]	[25.1,41.1]
A	E	[0.9,1.1]	[81.5,99.6]	[6.6,11.9]
A	E	[1.9,2.1]	[81.5,99.6]	[13.9,22.6]
A	E	[1.9,2.1]	[81.5,99.6]	[13.9,22.6]
A	E	[2.9,3.1]	[81.5,99.6]	[21.3,33.4]
B	C	[2.9,3.1]	[48.1,58.7]	[11.9,20.9]
E	A	[0.4,0.6]	[81.5,99.6]	[2.7,7.0]

5. Discussion

The example has assumed that many measurements are taken. In practice this does not happen, for example the pallets being moved from E to A are unlikely to be weighed, and often the exact number of empty pallets carried is not known, and the work involved in measuring them is not seen as worthwhile. A common way to handle this particular case is to use a default value such as $0.5\,\mathrm{t}$ for every return of empty pallets regardless of the number returned. The use of interval arithmetic allows a different way to handle this. Instead of requiring an exact weight of all the pallets it can provide an interval for the weight from simpler measurements. A driver could look at the pile of pallets loaded into his vehicle and say *"There are between 20 and 40 pallets"* without having to count each one. This could be combined with an interval for the weight of an empty pallet of between $15\,\mathrm{kg}$ and $30\,\mathrm{kg}$ to obtain an interval for the weight of all the pallets through interval arithmetic. i.e. $[0.015\,\mathrm{t}, 0.030\,\mathrm{t}] \cdot [20, 40] = [0.3\,\mathrm{t}, 1.2\,\mathrm{t}]$. The interval $[0.3\,\mathrm{t}, 1.2\,\mathrm{t}]$ gives an idea of the uncertainty in the weight of the pallets that the default value of $0.5\,\mathrm{t}$ does not. Interval arithmetic gives the logistic operator the ability to decide how much effort they wish to put into measuring for the amount of certainty in the result. They could make a blanket statement that pallet returns are between one and 100 pallets, this reduces the paperwork but increases uncertainty. Equally an organisation looking to decrease the uncertainty could look into their own pallets and discover that the type of pallets they use have weights between $20\,\mathrm{kg}$ and $25\,\mathrm{kg}$ or count every pallet. Using interval arithmetic allows organisations to decide how to invest into their data given the uncertainty in the data they gather.

6. Conclusion

An interval arithmetic based approach to quantify uncertainty in the logistics industry was set out based on formulae from ISO 14083:2023. A two part exemplar scenario was presented where the uncertainty of the variables in those formula was applied after being characterised and discussed. The proposed interval arithmetic approach to uncertainty gives the best possible bounds based on the information used. Whilst the example scenario focused on road freight, this methodology is applicable across the logistic industry and can be used to numerically quantify emissions uncertainty where the core data is available but more complex analytical methods are impractical.

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