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# Dynamic Predictive Maintenance Strategy based on a Time Transformation Method

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An efficient and cost-effective maintenance strategy is key to ensuring availability and performance of industrial assets. Conventional time-based scheduled maintenance often fails to achieve this goal, especially as it is static: it does not take into account asset condition evolution nor evolution in operational conditions over time. Here, we propose a predictive maintenance strategy which is dynamic, based on asset remaining useful life (RUL), and risk-informed. Building on a recently introduced time transformation that facilitates RUL estimation, this work develops a simulation framework that enables implementation, testing, and optimization of a dynamic maintenance strategy for a fleet of identical assets. RUL estimates are continuously updated, and the timing of the next maintenance intervention is optimized such that the probability of failure before that time does not exceed a specified bound. Alternatively, a warning threshold and the probability bound can be selected to minimize the total expected cost of preventive maintenance actions and failures (the latter including corrective maintenance and failure consequences). The proposed dynamic strategy is benchmarked against conventional (static) scheduled maintenance strategies. The cost of the scheduled preventive maintenance policy is compared with that of the predictive maintenance policy. The study also examines the effects of imperfect maintenance. A simplified yet realistic example is presented to demonstrate the applicability of the method.

*Keywords*: Remaining Useful Life, Dynamic Maintenance, Degradation, Cost Optimization, Time Transformation

## 1. Introduction

Prognostics and Health Management (PHM) refers to a broad set of methods and algorithms for detecting anomalies and degradations, diagnosing their root causes, and forecasting how they will evolve over time. Ultimately, one of the goals is to devise maintenance policies that guarantee availability and are cost effective in the assets' utilization context, and which can be adapted dynamically when that context evolves.

### 1.1. *State of the Art*

Considerable research has been devoted to predicting the Remaining Useful Life (RUL) of assets. However, a central objective of Prognostics and Health Management (PHM) is not only to estimate RUL but also to translate such predictions into cost-effective maintenance decisions Zio (2022). Determining maintenance policies in this context is challenging because the problem involves decision-making under uncertainty, as RUL predictions are inherently uncertain and operating conditions often vary over time. Moreover, maintenance planning is inherently sequential since the future degradation state of an asset depends on previous maintenance actions that were themselves based on earlier prognostic information. These characteristics make the optimization of maintenance strategies complex Nachlas (2017), often requiring the evaluation of conditional expectations and assumptions regarding future mission profiles, which typically leads to computationally intensive solution approaches. Various methods have therefore been proposed in the literature, including stochastic degradation model approaches and Markov decision processes Huynh et al. (2019), as well as more recent data-driven approaches leveraging deep learning techniques Fink et al. (2020). In parallel, models accounting for imperfect maintenance have been extensively studied since the seminal work of Kijima (1989) and subsequent developments Doyen and Gaudoin (2004). Since then, the notion of imperfect repair or maintenance has been extended to degrading systems where the efficiency of a preventive intervention can be characterized by an ”arithmetic reduction of degradation” (ARD) model Wang et al. (2021). In this context, the notion of a health index has emerged as a useful abstraction for representing the degradation state of an asset. A health index provides a continuous indicator of asset condition, facilitates the interpretation of degradation processes, and helps establish links between data-driven prognostics and physics-based models Dersin et al. (2025),Arias Chao et al. (2022).

### 1.2. *Proposed contribution and structure of the paper*

Our contribution here consists mainly in the use of a time transformation Dersin and Rocchetta (2026); Dersin (2023) to estimate the remaining useful life, based on field data. The goal of this paper is to describe a maintenance policy adapted to individual assets and evolving dynamically with the health state of the asset, as assessed by a health index that evolves from a perfect state (value 1) to failure (value 0) as degradation sets in and worsens. A warning threshold is selected and, when the health index reaches that threshold, a date is defined for the next preventive maintenance operation with the aim to perform that operation before a failure occurs. More precisely, it is desired to control failure risk: the next preventive maintenance date is defined in such a way as to bound the failure probability by a predefined value. The determination of that next maintenance date is facilitated by the time transformation introduced in Dersin and Rocchetta (2026),Dersin (2023), which makes a closed-form solution possible. Then the expected cost of maintenance per unit of time is calculated as a function of the two decision parameters: warning threshold and failure risk. It can even be minimized by a judicious choice of those parameters. The rest of the paper is structured as follows.

In Section 2, an overview of the methodology, determination of the next preventive maintenance, and description of the health index time evolution are provided. The simulation experiment is described in Section 3. Numerical results are reported in Section 4 for a range of threshold values and probability bounds, along with sensitivity analyses, and a comparison against a policy that relaxes the perfect-maintenance assumption. Section 5 closes with conclusions and suggestions for future research.

## 2. The proposed methodology

### 2.1. *Overview*

The proposed methodology is summarized in Figure 1. The health of the asset to be maintained is measured by a health index (HI), which evolves from a value 1 when the health of the asset is perfect to a value 0 when the function of the asset has been lost, i.e., a failure has occurred. Under the assumption of perfect maintenance, any preventive or corrective maintenance intervention resets the HI to 1. That is, maintenance restores the asset to an ”as-good-as-new” condition, after which degradation starts anew.

The general maintenance strategy, which belongs to the family of ”quantile-based inspection and repair policies” (Jardine et al. (2006)), is characterized by two parameters: a warning threshold

$\tau$ and a probability $\alpha$. When the health index first crosses the threshold $\tau$ (at time $t$, say), a preventive maintenance operation is scheduled to take place after a certain time period $s$ has elapsed. This time period is calculated in such a way that the probability of a failure occurring during the interval ($t$,$t+s$) does not exceed $\alpha$. Thus, with a probability at most $\alpha$, the health index reaches $0$ before time $t+s$; in which case a corrective maintenance operation takes place when HI reaches $0$. The probability $\alpha$ measures the risk of failure that the decision-maker is willing to take. The time to perform maintenance is considered negligible with the respect to the time between failures. For a given value of $\alpha$, the goal is to select the parameters $\tau$ and $s$ in such a way that $s$ is as large as possible while keeping the probability of a failure in ($t$,$t+s$) no greater than $\alpha$. If the statistical behavior of the $HI$ can be characterized, it will be possible to simulate the HI trajectories for a fleet or an asset. If unit preventive and corrective maintenance costs are known, the total maintenance cost (preventive and corrective) over a time horizon can be simulated. Ultimately, it will be possible to choose the threshold $\tau$ and the probability $\alpha$ to minimize the total expected cost over a time horizon. In what follows, therefore, we examine successively (i) how to determine the optimal time-to-next-preventive maintenance $s$ after threshold crossing (Section2.2) and (ii) how HI behavior over time can be modeled (Subsection2.3).

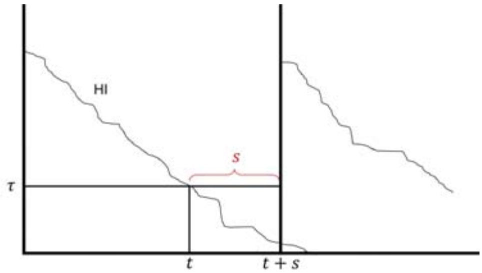

Fig. 1. Process of degradation and maintenance (assuming perfect maintenance) where the time-to-next maintenance $s$ is scheduled after the $HI$ first crosses the warning threshold $\tau$.

## 2.2. *Determining next maintenance operation*

One key element is, after the time $t$ at which the health index $HI$ crosses the chosen threshold $\tau$, to determine the time $s$ to the next preventive maintenance in such a way that the probability of failure before $t+s$ does not exceed a given probability $\alpha$. To that end, the time transformation introduced in Dersin and Rocchetta (2026) is used. It was shown there that there is always a time transformation $g(t)$ which transforms the physical time into a ”transformed” time for which the time to failure follows a Hall-Wellner distribution, so that the Mean Residual Life (MRL) is a linear function of (transformed) time and confidence intervals for the RUL can be expressed in closed form. The time transformation is expressed in terms of the reliability function $R(t)$ as follows:

$$g(t) = \frac{\mu}{k}\left(1 - R(t)^{\frac{k}{1-k}}\right), \tag{1}$$

where the parameter $\mu$ is the expectation of the time-to-failure (TTF), i.e., the MTTF, and the parameter $k$ is obtained from the coefficient of variation of TTF:

$$k = \frac{1 - cv^2}{1 + cv^2} \tag{2}$$

with the coefficient of variation $cv = \frac{\sigma}{\mu}$. Then, inverting that time transformation converts confidence intervals back to the physical time. Here, we need a one-sided confidence interval since we are only interested in $s$, a lower bound for the RUL (stating that failure will not occur earlier than a certain date). The following equation, derived from Dersin and Rocchetta (2026), is thus used in order to calculate that bound, at the $(1-\alpha)$ confidence level:

$$g(s) = \left(\frac{\mu}{k} - g(t)\right)\left[1 - (1-\alpha)^{\frac{k}{1-k}}\right] \tag{3}$$

In the above equation, $t$ denotes the first passage time of HI at level $\tau$, the first time when HI reaches the value $\tau$. Thus, $t = t(\tau)$. The transformation $g$ is an increasing function that ranges from 0 to $\mu/k$ as its argument varies from 0 to infinity. From Eq. (3), $s$ is obtained by inverting the function $g$ (when the latter is strictly increasing, which is true in most cases) i.e.,

$$s = g^{-1}\left(A(\tau)\left[1-(1-\alpha)^{\frac{k}{1-k}}\right]\right) \quad (4)$$

where $A(\tau) = (\mu/k - g(t(\tau)))$. This expression shows that $s$ is a function of the two decision parameters, $\tau$ and $\alpha$. It can be seen that $s$ is an increasing function of the threshold $\tau$ and the probability $\alpha$. Indeed, a higher value of $\tau$ implies a lower value of $t$ (the threshold is reached earlier) and therefore a higher value of $A(\tau)$ and therefore of the right-hand side of Eq. (4). On the other hand, the right-hand side is also an increasing function of $\alpha$: a higher probability $\alpha$ naturally leads to a longer maintenance interval, $s$.

### 2.3. *Degradation models*

Now, some assumptions are made about the HI curve that describes asset degradation. The HI curve can be arbitrary (non-parametric) or it can be modeled by a parametric stochastic model, for instance, the one studied in Dersin et al. (2025): This parametric model, which has been validated on some synthetic engine dataset (N-CMAPSS) Arias Chao et al. (2021), is convenient for illustration:

$$HI(t) = 1 - bt^p, \quad (5)$$

where b is a Fréchet random variable, and the p exponent is greater than 1. Such a model has been validated on field data, and it has been shown Dersin et al. (2025) that such an expression for the HI implies that the time to failure – the time for the $HI$ to reach level $0$ – is Weibull-distributed, and the parameters of the Weibull distribution are derived from those of the Fréchet variable $b$ and the exponent $p$. Alternative models, based on the Gamma distribution, for instance, or Wiener and Gamma processes, have also been validated on other field data.

### 2.4. *Maintenance Analysis and Dynamic maintenance optimization*

On the basis of subsections 2.2 and 2.3, an interactive algorithm has been constructed and implemented in Python; it can be described by the following steps:

**Step 1**. Input the HI model and the number of assets in the fleet.
**Step 2**. For each asset of the fleet, simulate the HI evolution until 0 according to the model.
**Step 3**. From the simulated data, estimate the g time transformation function, including the $\mu$ and $k$ parameters as per Dersin and Rocchetta (2026).
**Step 4**. Apply Equation (4) of Subsection 2.2 to determine the next preventive maintenance time after hitting the threshold $\tau$.
**Step 5**. Set HI to 1 (or a lower value in case of imperfect maintenance-see Subsection 2.5, and go back to Step 2.

Note that simulation is required only for Step 2: the computations in the other steps use closed-form expressions. The output is a set of HI trajectories, one for each asset, as illustrated in Figure 2. In a real-life application, the HI is estimated from measurements and does not have to be simulated; then the entire algorithm utilises closed-form expressions.

### 2.5. *Imperfect Maintenance*

In the case of imperfect maintenance, the maintenance does not restore the asset to an "as-good-as-new" state but rather to an intermediate state. Numerous models have been introduced in the literature to model that situation; in particular, the ARD model Wang et al. (2021). The ARD model is used here and is described by the HI after maintenance, which is increased by a fraction ($\rho$) of the difference between the perfect state and the state just before maintenance as follows:

$$HI_a = HI_b + \rho \cdot (1 - HI_b) \quad (6)$$

The suffixes $a$ and $b$ denote after and before maintenance, respectively. The perfect maintenance case corresponds to $\rho = 1$, whilst $\rho = 0$ corresponds to minimal maintenance.

## 3. Experiments and use case

The methodology is applied to a population of 100 assets, first assuming perfect maintenance, with a Fréchet-based HI with the following parameters Dersin et al. (2025): $p = 3.36$, Weibull TTF with shape parameter $\beta = 7$ and scale $\eta = 77$ cycles.

A cost model assumes a unit cost, $C_c$, for corrective actions and $C_p$ for preventive maintenance actions. The total cost over a horizon of $H$ cycles is given as follows:

$$C_{total} = N_f * C_c + N_p * C_p \qquad (7)$$

where $N_f$ denotes the number of failures over the horizon H, and $N_p$ the number of preventive interventions. Optimization is carried out at the threshold $\tau$ for a given probability $\alpha$, and simultaneously on $\tau$ and $\alpha$ by simulation of a large number of trajectories. The mean and standard deviation of the total cost over a given horizon are estimated. A two-stage grid-search optimization is used. In the first stage, the $(\tau, \alpha)$ space is explored using a coarse grid over a user-defined range; in the second stage, a finer grid is used to search for a smaller region identified in Stage 1. One can also let the horizon length tend to infinity and compare asymptotic costs per unit time. In practice, here the horizon $H$ is set to 1000 cycles. Since there are 100 assets, the average cost per asset per cycle is obtained by dividing the figures in the Tables by 100000.

## 4. Results

The results are summarized in Table 1 for several values of the threshold $\tau \in [0.01, 0.9]$, and the probability bound $\alpha \in [0.01, 0.20]$. The ratio of unit corrective cost to unit preventive cost is assumed to be $C_c = 10C_p$, i.e., a failure costs 10 times as much as a preventive maintenance operation. Between parentheses is indicated the percentage of the total cost accounted for by corrective maintenance. Figure 2 shows HI trajectories corresponding to the perfect maintenance case.

### 4.1. *Sensitivity analysis*

Then, the same study is carried out for a lower $\frac{C_c}{C_p} = 2$, and the results are summarized in Table 2 and shown in Figure 4. It is observed that the optimal ($\tau$,$\alpha$) pair is shifted, from $\tau$ equal to $0.2$ in the first case to $\tau$ equal to $0.1$ in the second case. Since in the second case avoiding failures is relatively less important, it is more advantageous to lower the threshold $\tau$ and reduce the amount of preventive maintenance. The results of the cost optimization algorithm are illustrated in Figure 3 by contour plots. Subsequently, the same analysis has been conducted (assuming $C_c = 10\ C_p$) in the imperfect maintenance case, assuming a maintenance efficiency factor $\rho$ equal to 0.8 (the results are shown in the right-panel part of Table 1). The optimum is reached for the same ($\tau$, $\alpha$) combination: $\tau = 0.2$ and $\alpha = 0.01$, as in the perfect maintenance case, but the costs are much higher. This is because maintenance operations restore the HI to a level less than 1, and it sub-

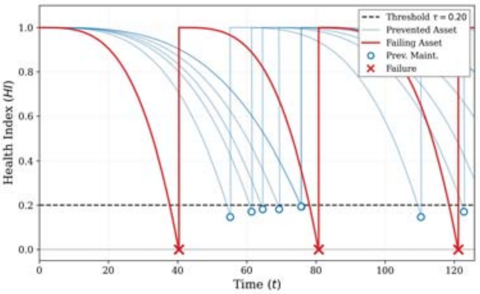

Fig. 2. After passing threshold $\tau$, the respective $s$ is calculated, and the asset is scheduled for perfect maintenance ($\rho = 1.0$). Blue lines indicate assets successfully maintained before failure, whereas red lines indicate assets that failed prior to the scheduled intervention at $t + s$

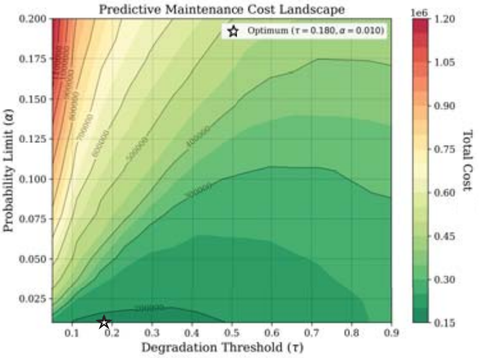

Fig. 3. Contour plot for different costs depending on degradation threshold $\tau$ and probability limit $\alpha$. The optimum (star-shaped marker) is found at $\tau = 0.2$ and $\alpha = 0.01$ for $\rho = 1.0$.

Table 1. Comparison of perfect and imperfect Maintenance for $C_c = 10\ C_p$. The resulting total costs $C_{total}$ are expressed in $\times 10^3$ monetary units, with a yellow cell highlighting the optimal combination of threshold ($\tau$) and accepted risk ($\alpha$). The percentages indicate the total cost for corrective maintenance.

| | Perfect ($\rho = 1$) | | | | Imperfect ($\rho = 0.8$) | |
|---|---|---|---|---|---|---|
| $\tau$ | $\alpha = 0.01$ | $\alpha = 0.05$ | $\alpha = 0.1$ | $\alpha = 0.2$ | $\alpha = 0.01$ | $\alpha = 0.05$ |
| 0.9 | 260.9 (15%) | 278 (33%) | 344 (55%) | 497 (75%) | 683 (15%) | - |
| 0.8 | 241 (16%) | 267 (35%) | 342 (56%) | 498 (76%) | 623 (16%) | - |
| 0.7 | 225 (17.5%) | 258 (36%) | 342 (58%) | 510 (77%) | - | - |
| 0.6 | 212 (18.4%) | 253 (38%) | 348 (60%) | 535 (79%) | - | - |
| 0.5 | 203 (20%) | 254 (41%) | 370 (65%) | 563 (82%) | - | - |
| 0.4 | 196 (21%) | - | - | - | 507 (21%) | - |
| 0.3 | 192 (23%) | - | - | - | 499 (24%) | - |
| 0.2 | 191 (26%) | 374 (68%) | - | - | 492 (26%) | 980 (69%) |
| 0.1 | 213 (37%) | 359 (78%) | - | - | 557 (28%) | 1350 (82%) |

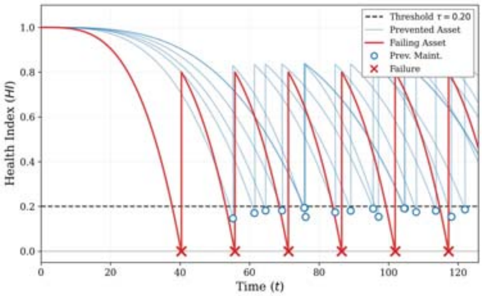

Fig. 4. After passing threshold $\tau$, the respective $s$ is calculated, and the asset is scheduled for imperfect maintenance ($\rho = 0.8$). Blue lines indicate assets successfully maintained before failure, whereas red lines indicate assets that failed prior to the scheduled intervention at $t + s$

sequently reaches the threshold sooner; therefore, more preventive interventions and more failures occur. In practice, most maintenance operations are imperfect, and assuming perfect maintenance is usually quite optimistic. A refined model would assume different efficiency levels across operations, for example, by specifying a probability of perfect maintenance.

### 4.2. *Comparison with scheduled maintenance policy*

In the scheduled maintenance policy, only one degree of freedom exists for decision making: the maintenance interval $T$. The classical approach Nachlas (2017),Dersin (2023) relies on the renewal function, which provides the average number of failures during a maintenance interval, so that the length of the maintenance interval can be selected to minimize the sum of corrective and preventive maintenance cost. In (Dersin (2023), Ch.9), an expression has been given for the renewal function of a Hall-Wellner distribution, which can be translated into the renewal function for the initial distribution via the time transformation.

However, here, we also use the same heuristic method as for the predictive maintenance strategy: simulating a large number of HI trajectories. This approach has the advantage that it can also be used in case of imperfect maintenance, where the sequence of inter-failure times is not a renewal process. Even for low values of the corrective-to-preventive cost ratio, the predictive maintenance strategy entails a lower cost than the scheduled one (presumably because two degrees of freedom are available, versus one for the scheduled maintenance policy). For instance, in the case $C_c = 10\ C_p$, the optimal scheduled maintenance cost, assuming perfect maintenance, is 285000 (resulting from a maintenance interval $T = 60$ h), to be compared with 191000 for the optimal predictive maintenance strategy. For imperfect maintenance with $\rho = 0.8$, these figures become 603000 and 492000, respectively. The cost reduction achieved by the predictive maintenance policy increases as the corrective-to-preventive cost ratio rises. This occurs because predictive maintenance is more

Table 2. Comparison of perfect and imperfect maintenance for $C_c = 2\ C_p$. The resulting total costs $C_{total}$ are expressed in $\times 10^3$ monetary units, with a yellow cell highlighting the optimal combination of threshold ($\tau$) and accepted risk ($\alpha$). The percentages indicate the total cost for corrective maintenance.

| | Perfect ($\rho = 1$) | | | Imperfect ($\rho = 0.8$) |
|---|---|---|---|---|
| $\tau$ | $\alpha = 0.01$ | $\alpha = 0.05$ | $\alpha = 0.1$ | $\alpha = 0.01$ |
| 0.9 | 229 (3%) | 204 (9%) | 192 (20%) | 601 (3%) |
| 0.8 | 210 (4%) | 194 (10%) | 189 (20%) | 546 (4%) |
| 0.7 | 193 (4%) | 184 (10%) | 185 (21%) | 502 (4%) |
| 0.6 | 181 (4.3%) | 176 (11%) | 180 (23%) | 468 (4.3%) |
| 0.5 | 172 (5%) | 170 (13%) | 178 (27%) | 442 (4.8%) |
| 0.4 | 163 (5%) | 166 (16%) | 178 (32%) | 421 (5%) |
| 0.3 | 157 (6%) | 166 (22%) | 181 (39%) | 405 (5.8%) |
| 0.2 | 152 (6%) | 169 (30%) | 187 (48%) | 390 (6.5%) |
| 0.1 | 150 (11%) | 181 (45%) | 212 (69%) | 386 (11%) |
| 0.05 | 158 (23%) | 206 (65%) | 241 (85%) | 409 (24%) |

effective at preventing failures, which becomes increasingly valuable when failures are relatively expensive

## 5. Conclusions and discussion

In this paper, we have capitalized on (i) the time transformation introduced earlier Dersin and Rocchetta (2026) and (ii) the use of health indices, as described in Dersin et al. (2025). We use this framework to compare several predictive maintenance strategies with each other and with scheduled maintenance, and to search for an optimal choice of maintenance parameters. The approach provides a useful support to maintenance decision making and requires only a minimal set of assumptions. It describes quantitatively the impact of the ratio of unit corrective to unit preventive maintenance cost, as well as the impact of imperfect maintenance, on both optimal decisions and minimum total cost, as well as the gain resulting from transitioning from scheduled to predictive maintenance. That gain can then be put into perspective with the investments necessary for condition monitoring and HI assessment. The work presented here can lay the foundation for future research, which may explore, for instance, time-varying degradation speed and systematically quantify prediction uncertainty. Equation (4) confirms that the smaller the probability $\alpha$, the shorter the next maintenance interval $s$ (almost by definition); and also, the higher the parameter k, the smaller $s$, which is to be expected since $k$ measures the rate of degradation. These results are general and do not depend on any specific assumptions about the evolution of the health index, although our method makes it possible to quantify them. In real-world applications where HI values are observed, there is no need to simulate HI evolution; instead, the dates of preventive interventions can be determined through the time transformation. Further work may consider:

- more general models for health index evolution, including both parametric and non-parametric approaches;
- the variability of operating conditions and degradation processes over time; and
- the investigation of a broader range of real-world case studies.

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