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A Reliability Evaluation Method for Binary-Node Multi-State Networks Considering Node–Component State Dependence

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Reliability evaluation of multi-state networks is fundamental to the design, operation, and optimization of complex systems. Traditional studies assume that both nodes and components possess multi-state capacities and behave independently, enabling node–component equivalence transformations to simplify reliability computation. However, for practically relevant Binary-Node Multi-State Networks (BN-MSNs), where nodes operate in strictly binary states (normal/failed) and all component capacities fully depend on incident node, such transformations are no longer applicable. In BN-MSNs, any node failure forces immediate capacity loss for all adjacent components, significantly increasing modeling and computational complexity. To address this challenge, this paper proposes an extended modeling framework and an indirect reliability evaluation algorithm tailored for BN-MSNs. An enhanced capacity model is first developed to embed the "node failure induces component failure" dependency mechanism. Based on this model, network reliability is decomposed into reliabilities of subgraphs corresponding to all possible node-state combinations, and overall reliability is reconstructed via the law of total probability. To mitigate combinatorial explosion of node-state space, a pruning theorem for node-state combinations is established, effectively eliminating subgraphs that do not require further reliability analysis. Building upon this reduced search space, the method integrates multi-state minimal cut enumeration with probabilistic union computation to efficiently derive BN-MSN reliability. Experimental results demonstrate that the proposed method significantly reduces computational complexity versus traditional methods and consistently outperforms ablation benchmarks, validating the superior efficiency of integrated pruning and modeling strategies.

Keywords: Multi-state network (MSN), Binary-node multi-state network (BN-MSN), Node–component dependence, Multi-state minimal cut, Reliability evaluation algorithm.

1. Introduction

With the increasing complexity of modern infrastructures, reliability evaluation for critical systems such as communication, power, and computing networks has become a research hotspot for safeguarding societal operations. As a core step in complex system design, it also serves as the foundation for monitoring and performance optimization. In traditional network reliability evaluation, multi-state networks (MSNs) can meticulously characterize component capacity across different degradation levels, demonstrating higher engineering value than conventional binary-state models. In classical MSN research, it is commonly assumed that both nodes and components exhibit similar multi-state characteristics. Jane et al. (1993) defined two-terminal MSN reliability as the probability that the maximum flow from source to sink meets a given demand d. Based on this, extensive studies have investigated evaluation methods using multi-state minimal paths (d-MPs) or minimal cuts (d-MCs) (Bai et al., 2018, 2023; Kozyra, 2023; Lin, 2001, 2007; Niu et al., 2024; Zhang et al., 2024). Most approaches assume component failure independence, enabling node–component equivalence transformations to map node reliability into virtual components, thus reducing hybrid models to pure component-based MSNs. However, in practical engineering systems like integrated circuits and distributed computing clusters, a distinct node–component dependency exists: binary-state nodes exert 'veto power' over connected components. Once a node fails, associated component capacities immediately drop to zero, leading to local transmission interruption. This special structure, featuring binary-state nodes and multi-state components, is termed the Binary-Node Multi-State Network (BN-MSN) in this paper.

Regarding reliability analysis of such networks, Hänsler (1972) achieved exact evaluation for binary-state networks with node–component joint failures using recursive subnet combination, assuming independence between node and component failures. Aggarwal et al. (2003) proposed transforming node failures into equivalent component failures to reuse classical methods. However, Lin (2001) showed that such transformations are limited to binary-state systems. For BN-MSNs, node failures induce dependency among components, invalidating the independence assumption and transforming the problem into a complex conditional probability combination. Moreover, as this problem is NP-hard, traditional full-state enumeration based on maximum flow (Frank and Hakimi, 1965) suffers from exponential complexity growth, rendering it impractical for large-scale networks. While traditional networks can conceptually be mapped to Fault Trees (FT), applying standard FT quantification algorithms to BN-MSNs presents significant computational challenges. The strict node-component dependency in BN-MSNs requires complex Multi-state or Dynamic Fault Trees. Since standard FT algorithms are primarily designed for independent events, quantifying BN-MSNs via FT mapping typically leads to state-space explosion. Thus, developing specialized network-based algorithms provides a more direct and computationally viable solution.

To address these limitations, this paper proposes an efficient reliability evaluation method for BN-MSNs. First, an enhanced capacity model is developed by embedding node failure-induced component failures directly into component state vectors using logical operators, thereby explicitly capturing node–component dependency. Second, the global reliability is decomposed into subgraph reliability problems under specific node state combinations using recursive decomposition and total probability theory. To mitigate the exponential growth of the node state space, a pruning theorem is derived based on network flow monotonicity and source–sink connectivity, which eliminates subgraphs that do not contribute to overall reliability. Within the reduced search space, the proposed method integrates d-MC enumeration techniques (Niu and Xu, 2019; Zhang et al., 2024) with the Recursive Sum of Disjoint Products (RSDP) algorithm (Bai et al., 2015; Zuo et al., 2007) to achieve accurate and efficient indirect reliability evaluation.

Numerical experimental results demonstrate that the proposed reliability evaluation method significantly outperforms the enumeration method (Frank and Hakimi, 1965) and ablation experimental versions without complete pruning strategies (e.g., the comparison schemes without pruning, with only Criterion 1, or with only Criterion 2) in computational efficiency while maintaining solution accuracy. This study not only theoretically extends the modeling boundary of MSN reliability but also provides an efficient computational tool for reliability evaluation and optimal decision-making of complex network systems through the systematic "decomposition-pruning-integration" strategy.

2. Modeling and Problem Definition

Consider a directed network denoted as $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E}, \boldsymbol{W})$, where $\boldsymbol{V}$ is the node set with $n = |\boldsymbol{V}|$, the source node s and sink node t are elements of $\boldsymbol{V}$. $\boldsymbol{E}$ represents the component set with $m = |\boldsymbol{E}|$, where each component is denoted as e_i for $1 \le i \le m$. $\boldsymbol{x} = (x_1, x_2, \dots, x_m)$ denotes the network state vector reflecting the current states

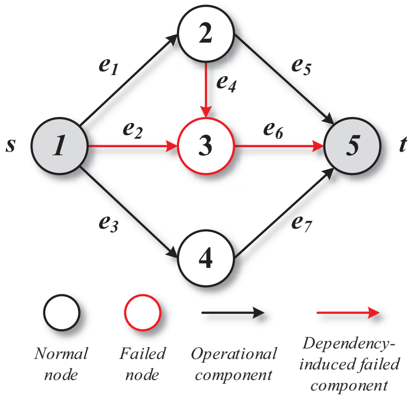

Fig. 1. Schematic diagram of BN-MSN.

of all components. $\boldsymbol{W} = (W_1, W_2, \dots, W_m)$ is the maximum state vector composed of the maximum state of each component. Let $\boldsymbol{b_i} = (b_{i1}, b_{i2}, \dots, b_{iw_i})$ denote the basic state space vector of component e_i. In a BN-MSN, each node has a binary state $s_v \in \{0,1\}$ representing normal operation or failure: the probability of a node being in the normal state ($s_v = 1$) is defined as p_v; when $s_v = 0$, the node fails, and the states of all its adjacent components are reduced to 0 immediately. The effective state of component $e_j = (u, v) \in \boldsymbol{E}$ depends on its basic state $\boldsymbol{b}_j$and the states of its two endpoints (nodes u and v), which is defined as follows:

$$\tilde{\boldsymbol{b}}_j = \begin{cases} \boldsymbol{b}_j & if\ s_u = s_v = 1 \\ 0 & otherwise \end{cases} \qquad (1)$$

The graphical representation of the BN-MSN is illustrated in Fig. 1.

Given a flow demand of $d+1$ to be transmitted from s to t in $\boldsymbol{G}$, the reliability $R_{d+1}$of the BN-MSN is defined as the probability that the transmission capacity from s to t is no less than $d+1$, as shown in Eq. (2).

$$R_{d+1} = Pr(\varphi(\boldsymbol{x}) \geq d+1) \qquad (2)$$

Herein, $\boldsymbol{x}$ denotes the current component state vector of the BN-MSN, and $\varphi(\boldsymbol{x})$ is the network structure function representing the maximum flow under state $\boldsymbol{x}$.

3. A BN-MSN Reliability Evaluation Method Based on Total Probability Decomposition

3.1. *Theoretical analysis*

To address the computational bottleneck of the state enumeration method in BN-MSN reliability evaluation, this paper proposes an efficient solution strategy based on total probability decomposition and indirect evaluation. First, drawing on the recursive decomposition idea proposed by Hänsler (1972), the reliability solution task of the original network is expanded via total probability with respect to node state combinations. On this basis, with reference to the indirect evaluation process for traditional MSNs (Yeh, 2004), an exact calculation of reliability R_{d+1} is achieved by identifying all d-MCs meeting the demand $d+1$ and computing the union probability of their associated events.

To avoid state space explosion arising from explicit enumeration of combinations of node states s_v and component state vectors $\boldsymbol{b}_j$, this paper adopts a hierarchical decoupling search logic: first, enumerate global node state combinations to construct equivalent subgraphs; second, extract d-MCs and calculate subgraph reliability under subgraph constraints; finally, perform global aggregation to derive R_{d+1}. The decomposition process of the proposed solution is illustrated in Fig. 2.

All possible node state combinations in a BN-MSN are defined as $\boldsymbol{S} = (\boldsymbol{S}_1, \dots, \boldsymbol{S}_\rho)$, where $\boldsymbol{S}_i = (s_1, \dots, s_u, \dots, s_n)$ denotes a feasible value vector of node states, with the source node as s_1 and the sink node as s_n. Let $\boldsymbol{G}^{\boldsymbol{S}_i} = (\boldsymbol{V}^{\boldsymbol{S}_i}, \boldsymbol{E}^{\boldsymbol{S}_i})$ represent the induced subgraph under node state combination $\boldsymbol{S}_i$, where $\boldsymbol{V}^{\boldsymbol{S}_i} = \{v \in \boldsymbol{V}^P | s_v = 1\}$ is the current surviving node set with n_i, and $\boldsymbol{E}^{\boldsymbol{S}_i} = \{e_i = (u, v) \in \boldsymbol{E} | u, v \in \boldsymbol{V}^{\boldsymbol{S}_i}\}$ is the component set induced by the surviving node set with m_i. Let W_j denote the maximum state of e_i, and $\boldsymbol{W}^{\boldsymbol{S}_i} = (W_1, W_2, \dots, W_{m_i})$ be the maximum state vector of the subgraph. Given that the subgraph $\boldsymbol{G}^{\boldsymbol{S}_i}$ is in vector $\boldsymbol{W}^{\boldsymbol{S}_i}$, the maximum flow from s to t is defined as $\varphi(\boldsymbol{W}^{\boldsymbol{S}_i})$.

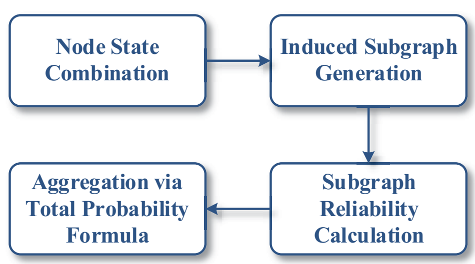

Fig. 2. Flowchart of the solution procedure.

According to the total probability formula, the reliability R_{d+1} of the BN-MSN can be expressed as the weighted sum of conditional reliabilities under all node state combinations:

$$R_{d+1} = \sum_{i=1}^{\rho} Pr(\boldsymbol{S}_i) \cdot Pr(\varphi(\boldsymbol{x}) \geq d+1|\boldsymbol{S}_i) \quad (3)$$

Herein, ρ denotes the number of possible node state combinations, and the prior probability of the occurrence of node state combination $\boldsymbol{S}_i$ is given by:

$$Pr(\boldsymbol{S}_i) = \prod_{j=1}^{n_i} p_j^{s_j}(1-p_j)^{1-s_j} \quad (4)$$

Given $\boldsymbol{S}_i$, the conditional probability that the subgraph meets the demand can be calculated via the *d*-MC set:

$$Pr(\varphi(\boldsymbol{x}) \geq d+1|\boldsymbol{S}_i) = 1 - Pr\left(\bigcup_{j=1}^{\sigma_i}\{\boldsymbol{x} \leq \boldsymbol{z}_{i,j}\}\right) \quad (5)$$

Where, $Pr(\boldsymbol{S}_i)$ denotes the occurrence probability of node state combination $\boldsymbol{S}_i$ in the BN-MSN; $Pr(\varphi(x) \geq d+1|\boldsymbol{S}_i)$ is the probability that the maximum flow from s to t in the network meets the demand $d+1$ under this node state combination; $\boldsymbol{z}_{i,j}$ represents the *j*-th *d*-MC of the subgraph $\boldsymbol{G}^{\boldsymbol{S}_i}$ under $\boldsymbol{S}_i$.

Through the aforementioned decomposition process, the original problem is transformed into a series of smaller-scale subgraph reliability evaluation problems, which can effectively reduce the complexity of reliability evaluation.

3.2. *Pruning Criteria*

In the aforementioned theoretical analysis, the complete set of network states is constituted by the Cartesian product of node and component state vectors. Since each node has two discrete states (normal and failed), the number of node-level state combinations reaches 2^n for a network with n nodes; such exponential growth leads to the dilemma of the curse of dimensionality and combinatorial explosion when handling large-scale networks. Thus, this section pre-prunes the state space using the developed pruning criteria, avoiding the high computational complexity caused by enumerating all node combinations.

Pruning Criteria (Theorem 1): For any $\boldsymbol{S}_i$, if either of the following conditions is satisfied, the subgraph $\boldsymbol{G}^{\boldsymbol{S}_i}$ derived from this combination can be pruned (i.e., there is no need to perform subsequent *d*-MC search):

- Boundary failure criterion: The combination is directly eliminated if the source node or sink node fails ($s_1 = 0$ or $s_n = 0$).
- Connectivity and flow criterion: The combination contributes nothing if the subgraph $\boldsymbol{G}^{\boldsymbol{S}_i}$ is disconnected or the maximum flow $\varphi(\boldsymbol{W}^{\boldsymbol{S}_i})$ under $\boldsymbol{W}^{\boldsymbol{S}_i}$ is less than or equal to d.

Proof: The two criteria are discussed separately below.

For Criterion 1, if $s_1 = 0$ or $s_n = 0$, i.e., the source node or sink node fails, then by definition, $s \notin \boldsymbol{V}^{\boldsymbol{S}_i}$ or $t \notin \boldsymbol{V}^{\boldsymbol{S}_i}$. The subgraph $\boldsymbol{G}^{\boldsymbol{S}_i}$ thus contains neither the source nor the sink node, which contradicts the definition of reliability and can therefore be pruned.

For Criterion 2, if nodes s and t are disconnected in subgraph $\boldsymbol{G}^{\boldsymbol{S}_i}$ or the maximum flow $\varphi(\boldsymbol{W}^{\boldsymbol{S}_i})$ under the maximum state vector $\boldsymbol{W}^{\boldsymbol{S}_i}$ satisfies $\varphi(\boldsymbol{W}^{\boldsymbol{S}_i}) \leq d$, this combination should be pruned. This is because the target probability to be calculated is $Pr(\varphi(\boldsymbol{x}) \geq d+1|\boldsymbol{S}_i) = 1 - Pr\left(\bigcup_{j=1}^{\sigma_i}\{\boldsymbol{x} \leq \boldsymbol{z}_{i,j}\}\right)$, which requires the network flow under a specific state to be at least $d+1$. The vector $\boldsymbol{W}^{\boldsymbol{S}_i}$ represents the physical upper bound of network capacity under $\boldsymbol{S}_i$, and any actual state vector $\boldsymbol{x}$ is necessarily constrained by this physical capacity (i.e., $\boldsymbol{x} \leq \boldsymbol{W}^{\boldsymbol{S}_i}$, so $\varphi(\boldsymbol{x}) \leq \varphi(\boldsymbol{W}^{\boldsymbol{S}_i})$). If $\varphi(\boldsymbol{W}^{\boldsymbol{S}_i}) \leq d$, then $\varphi(\boldsymbol{x}) \leq d < d+1$ holds invariably. This implies that under this $\boldsymbol{S}_i$, the network flow is always less than $d+1$ regardless of the random variation of $\boldsymbol{x}$. Therefore, the network cannot meet the reliability requirement under this state, and there is no need to perform further *d*-MC generation (Niu and Xu, 2019).

In conclusion, any node state combination $\boldsymbol{S}_i$ that satisfies either criterion can be safely pruned during the enumeration process.

3.3. *BN-MSN Reliability Evaluation Algorithm*

Grounded on the above discussions, the exact reliability evaluation algorithm flow, illustrative example, and time complexity analysis for BN-MSNs incorporating Theorem 1 are presented as follows.

3.3.1. *Algorithm Procedure*

The algorithm procedure is presented as follows.

Input: BN-MSN $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E}, \boldsymbol{W})$, component state vectors $\boldsymbol{b}_j$, probability set $\boldsymbol{P}_v$ of nodes in normal state, and network demand *d*.

Output: Network reliability R_{d+1}.

Step 1: Enumerate node states and perform simultaneous pruning. Generate node state combinations $\boldsymbol{S}_i = (s_1, \dots, s_u, \dots, s_n)$ one by one. For each $\boldsymbol{S}_i$, apply Theorem 1 to check if it satisfies the pruning criteria (boundary failure criterion, connectivity and flow criterion). If satisfied, skip this combination; otherwise, calculate its probability according to Eq. (4).

Step 2: Construct subgraphs and determine component states. For unpruned $\boldsymbol{S}_i$, construct the subgraph $\boldsymbol{G}^{\boldsymbol{S}_i} = (\boldsymbol{V}^{\boldsymbol{S}_i}, \boldsymbol{E}^{\boldsymbol{S}_i})$, and calculate the m_i effective component state vectors $\widetilde{\boldsymbol{b}}_j$ based on Eq. (1).

Step 3: Conduct *d*-MC generation. In subgraph $\boldsymbol{G}^{\boldsymbol{S}_i}$, search for *d*-MC $\boldsymbol{z}_{i,j}$ using the method proposed in Niu and Xu (2019).

Step 4: Calculate subgraph reliability. Based on the RSDP algorithm (Bai et al, 2015), compute the union probability of all *d*-MC associated events in the subgraph according to Eq. (5).

Step 5: Accumulate total reliability. For all unpruned $\boldsymbol{S}_i$, accumulate subgraph reliabilities according to Eq. (3) to obtain the reliability R_{d+1} of the BN-MSN.

3.3.2. *Illustrative example*

To demonstrate the execution process of the proposed algorithm step-by-step, the network illustrated in Fig. 1 is taken as an example. This network comprises 5 nodes and 7 components. The normal operation probability of all nodes is set to $p_v = 0.9$; all components share an identical state vector $\boldsymbol{b}_i = (0, 1, 2, 3)$, corresponding to a maximum capacity of $W_i = 3$, and their state distribution probability vectors are uniformly $\boldsymbol{p}_i = (0.25, 0.25, 0.25, 0.25)$. Assuming a network transmission demand of $d = 4$, the target is to solve for R_5.

- **Step 1: Node State Enumeration and Simultaneous Pruning**

For the network with 5 nodes, the original number of node-level state combinations totals $2^5 = 32$. In accordance with **Theorem 1**, online pruning is performed on these combinations:

1) **Boundary failure pruning:** If Node 1 or Node 5 fails (i.e., $s_1 = 0$ or $s_5 = 0$), the combination cannot form an effective transmission path. For instance, the state combination $\boldsymbol{S}_k = (0, 1, 1, 1, 1)$ is directly eliminated due to source node failure.
2) **Connectivity and flow pruning:** Consider the scenario where both Node 2 and Node 3 fail in Figure 1. When $s_2 = s_3 = 0$, all components dependent on these nodes $\{e_1, e_2, e_4, e_5, e_6\}$ are forced to fail. In this case, only $\{e_3, e_7\}$ remain in the subgraph. Maximum flow calculation shows that the theoretical maximum flow of the network is $\phi(\boldsymbol{W}^{\boldsymbol{S}_i}) = min(W_3, W_7) = 3 < d = 4$, so this combination contributes nothing and can be pruned.

- **Step 2: Subgraph Construction and Determination of Effective Component States**

A typical unpruned state is selected: all nodes are normal, i.e., $\boldsymbol{S}_i = (1, 1, 1, 1, 1)$. Under this node state combination, the subgraph $\boldsymbol{G}^{\boldsymbol{S}_i}$ is consistent with the original network structure, and its occurrence probability is $Pr(\boldsymbol{S}_i) = 0.9^5 = 0.59049$. Since the states of all endpoint nodes s_u and s_v are 1, the effective state of each component satisfies $\widetilde{\boldsymbol{b}}_j = \boldsymbol{b}_j$ according to Eq. (1), meaning all components retain their original multi-state distribution characteristics.

- **Step 3: *d*-MC Generation**

In the aforementioned subgraph $\boldsymbol{G}^{\boldsymbol{S}_i}$, we need to identify all 4-MCs. The set of 4-MCs generated via the method in Niu and Xu (2019) is provided below (partial display):

$$\{\boldsymbol{z}_{i,1} = (3,3,3,3,0,1,3), \boldsymbol{z}_{i,2} = (3,3,3,3,0,2,2),$$
$$\boldsymbol{z}_{i,3} = (3,3,3,3,0,3,1), \boldsymbol{z}_{i,4} = (3,3,3,3,1,0,3),$$
$$\boldsymbol{z}_{i,5} = (1,3,3,3,3,1,2), \boldsymbol{z}_{i,6} = (0,2,3,3,3,3,2),$$
$$\boldsymbol{z}_{i,7} = (3,0,3,1,3,3,3), \boldsymbol{z}_{i,8} = (3,2,0,0,2,3,3), \dots\}$$

- **Steps 4 & 5: Subgraph Reliability Calculation and Total Reliability Aggregation**

The RSDP algorithm (Bai et al, 2015) is applied to process the 4-MC set generated in **Step 3**.

1) Subgraph reliability: Calculate $1 - Pr(\cup\{\boldsymbol{x} \leq \boldsymbol{z}_{i,j}\})$. Under the fully normal state $\boldsymbol{S}_i$, the reliability of this subgraph meeting $d+1=5$ is calculated as $R_5^{S_i} = 0.143799$.
2) Network-wide aggregation: Repeat the above process for other valid node state combinations (e.g., only Node 2 fails, $\boldsymbol{S}_j = (1,0,1,1,1)$) and multiply by their respective state probabilities $Pr(\boldsymbol{S}_j)$.
3) Final result: By accumulating the reliability contributions of subgraphs induced by all unpruned combinations according to Eq. (3), the total reliability of the BN-MSN under $d=4$ is finally obtained as $R_5 = 0.090486$.

This example demonstrates that the pruning criteria drastically reduce the number of times the computationally expensive **Step 3** (d-MC generation) needs to be executed in **Step 1**, thereby significantly improving the algorithm's computational efficiency for complex networks.

3.3.3. *Time complexity*

This section conducts a theoretical analysis of the time complexity of the proposed BN-MSN reliability evaluation algorithm with pruning criteria. Via the two pruning criteria, the algorithm pre-eliminates a large number of node combinations with zero reliability contribution in **Step 1**. Let $\boldsymbol{\Omega}_{valid}$ denote the set of valid node state combinations after pruning, with its cardinality satisfying $|\boldsymbol{\Omega}_{valid}| \ll 2^n$. Thus, the proposed algorithm only needs to traverse $|\boldsymbol{\Omega}_{valid}|$ possible node state combinations. Its total time complexity mainly consists of two parts: d-MC generation for subgraphs and reliability calculation via the RSDP algorithm:

1) d-MC generation: The complexity of generating d-MCs for each subgraph i is $O(m\pi\sigma_i)$, where π denotes the complexity of the maximum flow algorithm.
2) Reliability calculation: Invoking the RSDP algorithm for each subgraph yields a complexity of $O(2^{(1-\alpha)\sigma_i})$, where $\alpha \in [0.2,0.4]$ is the computational optimization coefficient.

In conclusion, the total time complexity T_{total} of the exact BN-MSN reliability evaluation algorithm with pruning criteria can be expressed as:

$$T_{total} = \sum_{i=1}^{|\boldsymbol{\Omega}_{valid}|} O\left(m\pi\sigma_i + 2^{(1-\alpha)\sigma_i}\right) = O\left(|\boldsymbol{\Omega}_{valid}| \cdot 2^{(1-\alpha)\bar{\sigma}}\right)$$

Where $\bar{\sigma}$ represents the average number of d-MCs across all valid subgraphs.

4. Numerical experiments

This section selects four typical benchmark networks as illustrated in Fig. 4 (derived from Niu et al. (2017) and Xu et al. (2024) respectively) to verify the accuracy and efficiency of the proposed algorithm for BN-MSN reliability evaluation. The experiments assume consistent state distributions for all nodes and components across the networks, with detailed parameters provided in Table 1.

To comprehensively evaluate the contribution of the pruning criteria to computational efficiency, this study designs five algorithms for comparative experiments (i.e., ablation experiments) as follows:

1) SE (State Enumeration): The state enumeration method adopted as the baseline (Frank and Hakimi, 1965);
2) PM-NP (Proposed Method - No Pruning): The proposed method without enabling the pruning strategy;
3) PM-R1 (Proposed Method - Rule 1): The proposed method with only the boundary failure criterion enabled;
4) PM-R2 (Proposed Method - Rule 2): The proposed method with only the connectivity and flow criterion enabled;
5) PM-Full (Complete Proposed Method): The complete algorithm integrating dual pruning criteria.

All algorithms are implemented in Python 3.9 and executed on a personal computer equipped with an AMD Ryzen 7 5800H CPU (3.20 GHz) and 32 GB RAM. The experiments evaluate the consistency of the reliability results produced by different algorithms, along with their computational performance in terms of CPU runtime (denoted as T_1– T_5) and relative time ratios.

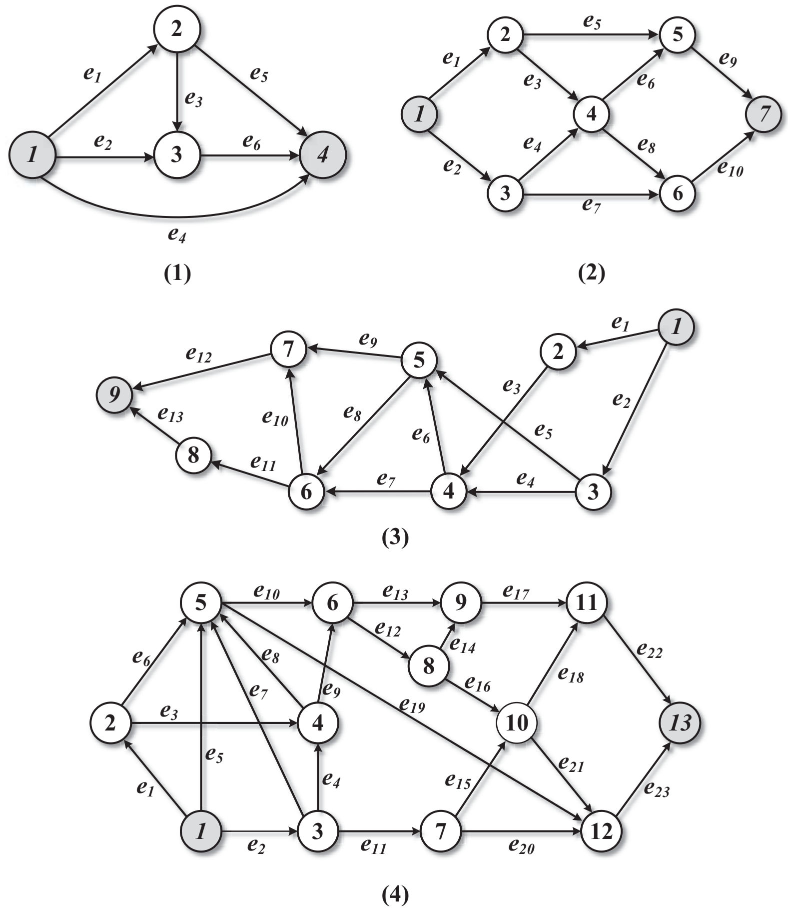

Fig. 3. Benchmark networks.

Table 1. Node and component state distribution of benchmark networks.

Network ID	Element type	State space vector	Probability Vector
Net (1)	Node	(0, 1)	(0.1, 0.9)
	Component	(0, 1, 2, 3, 4)	(0.20, 0.20, 0.20, 0.20, 0.20)
Net (2)	Node	(0, 1)	(0.2, 0.8)
	Component	(0, 1, 2, 3)	(0.25, 0.25, 0.25, 0.25)
Net (3)	Node	(0, 1)	(0.1, 0.9)
	Component	(0, 1, 2, 3)	(0.25, 0.25, 0.25, 0.25)
Net (4)	Node	(0, 1)	(0.2, 0.8)
	Component	(0, 1, 2)	(0.30, 0.30, 0.40)

Table 2. Computational time (s) and reliability of 5 methods on Net (1)

d	T_1	T_2	T_3	T_4	T_5	R_{d+1}
1	5.060	0.021	0.020	0.020	0.019	0.724986
2	4.673	0.054	0.059	0.052	0.051	0.632275
3	4.916	0.193	0.190	0.196	0.181	0.508981
4	4.840	0.423	0.437	0.438	0.421	0.363945
5	5.136	0.725	0.663	0.656	0.625	0.234638

Table 3. Computational time (s) and reliability of 5 methods on Net (2)

d	T_1	T_2	T_3	T_4	T_5	R_{d+1}
1	1425	0.498	0.446	0.485	0.445	0.189227
2	1401	3.001	3.033	2.772	2.636	0.068168
3	1434	2.932	2.941	2.692	2.626	0.015379
4	1376	1.524	1.555	1.531	1.457	0.002750
5	1440	0.417	0.427	0.358	0.349	0.000193

Table 4. Computational time (s) and reliability of 5 methods on Net (3)

d	T_1	T_2	T_3	T_4	T_5	R_{d+1}
1	>80000	3.481	3.189	3.024	3.018	0.189746
2		38.85	38.74	38.37	35.88	0.049233
3		110.3	109.7	109.6	109.3	0.006083
4		46.81	46.72	45.91	45.53	0.000441
5		4.500	4.510	3.681	3.346	0.000008

Table 5. Computational time (s) and reliability of 5 methods on Net (4)

d	T_1	T_2	T_3	T_4	T_5	R_{d+1}
0	>200000	90.88	91.83	83.85	82.64	0.306901
1		2701	2767	2699	2652	0.109845
2		11142	10974	11102	10896	0.015809
3		9851	9875	9949	9816	0.001405

As presented in Tables 2 to 5, the reliability results (R_{d+1}) obtained by the four improved methods proposed in this study (T_2 to T_5) are completely consistent with those of the traditional SE method across the four benchmark network scenarios. This finding fully verifies the accuracy of the proposed algorithm for BN-MSN reliability evaluation.

In terms of computational efficiency, the proposed method demonstrates remarkable superiority. Taking Net (2) as an example, when $d = 1$, the traditional enumeration method takes as long as 1425 s, while the improved method in this study only requires approximately 0.445 s, achieving an order-of-magnitude breakthrough. More importantly, in the more complex Net (4), the SE method fails to complete the solution within an effective time frame ($> 2 \times 10^5$ s) due to combinatorial explosion; in contrast, the proposed method yields accurate results within about 2652 s by introducing efficient pruning criteria. This indicates that the proposed method effectively overcomes the curse of dimensionality in large-scale BN-MSN reliability evaluation and exhibits excellent scalability and engineering applicability.

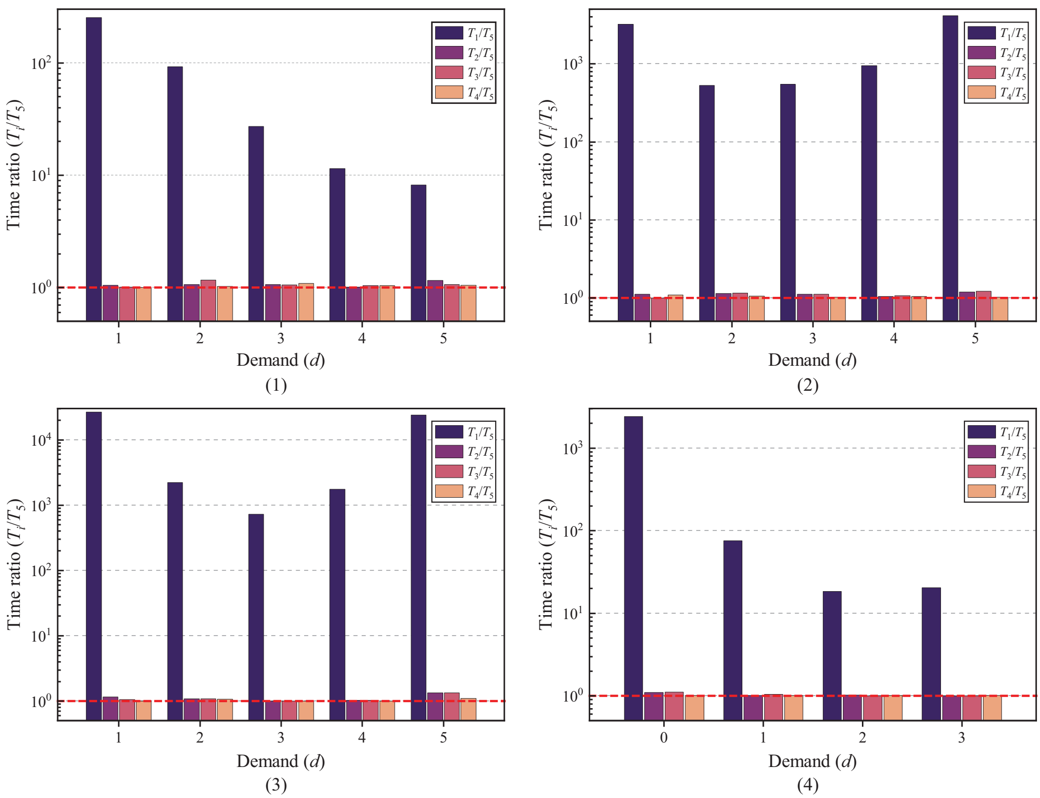

Fig. 4. Time ratios of four methods vs. the proposed PM-Full method.

Fig. 4 presents the computational time ratios (T_i/T_5) of the four comparative algorithms relative to the complete proposed method (PM-Full). The red horizontal dashed line in the figure represents the baseline with a ratio of 1, corresponding to the execution efficiency of PM-Full.

The experimental results indicate that the ratio values of all comparative algorithms are greater than 1, which quantitatively verifies the remarkable effectiveness of the pruning criteria proposed in **Section 3.2** in improving computational efficiency. In particular, the ratios of the state enumeration method (SE, T_1) to the proposed method reach the order of 10^2 or even 10^3 in multiple cases, demonstrating the enormous search space compression capability of the pruning strategy. Moreover, the time ratios of the methods without pruning or with only a single criterion (T_2, T_3, T_4) to T_5 remain consistently above the baseline. This strongly proves that the combined application of the boundary failure criterion and the connectivity and flow criterion achieves an optimal synergistic acceleration effect, which can minimize the computational cost of BN-MSN reliability evaluation.

5. Conclusions

This study proposes an efficient reliability evaluation method for Binary-Node Multi-State Networks (BN-MSNs) to address the complex node-component state dependence. By embedding the "node failure induces component failure" mechanism into an enhanced capacity model, the global reliability is decomposed into subgraph problems using total probability theory.

To overcome the combinatorial explosion of the node-state space, a pruning theorem is established based on boundary failure and connectivity-flow criteria. This strategy effectively eliminates multiple classes of subgraphs that do not contribute to reliability, significantly compressing the search space. Within the reduced space, the integration of multi-state minimal cut (*d*-MC) enumeration and the Recursive Sum of Disjoint Products (RSDP) algorithm ensures accurate calculation.

Numerical experiments on benchmark networks demonstrate that the proposed method maintains solution accuracy while significantly outperforming not only the traditional state enumeration method but also ablation versions lacking the complete pruning strategy in terms of computational efficiency. The method effectively mitigates the "curse of dimensionality," providing a robust theoretical extension and a practical tool for the reliability evaluation of complex network systems.

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