

Reliability Assessment of Multi-Span Masonry Arch Bridges Considering Geometrical Uncertainties

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Historical masonry arch bridges (MABs) are more than structural assets; they embody cultural identity, architectural heritage, and the construction knowledge of past generations. In Portugal, many metallic bridges were replaced by MABs, which today form an essential part of the national railway network. Due to their age and the cumulative effects of degradation, these structures are particularly vulnerable to both natural and anthropogenic hazards. The occurrence of such events may lead to disruptions in functionality, with significant consequences for mobility and infrastructure resilience. This study focuses on the structural assessment of MABs in Portugal, explicitly accounting for geometrical uncertainties associated with this bridge typology. The analysis is conducted using the LUSAS finite element analysis software package in combination with Grasshopper, which enables parameterization of the bridge geometry and its integration into a stochastic assessment framework. The reliability index is employed as the key performance indicator to evaluate structural performance. The proposed approach is demonstrated through an application based on the empirical design rules of a historical masonry arch bridge located in Portugal. The results highlight the influence of geometrical uncertainties on the reliability index and demonstrate the potential of the proposed framework to support decision-makers in the preservation and management of historical bridge assets.

Keywords: Surrogate models, Heritage structures, Finite element method, **Out-of-plane seismic capacity, Modal pushover** analysis.

1. Introduction

Masonry arch bridges (MABs) are among the most common bridge types in European railway networks, and many have already exceeded their original design life while remaining in service with limited maintenance needs (Silva 2022; Baron et al. 2024). Given the low redundancy of railway networks, the failure of a single bridge can cause major disruptions and significant indirect losses, underscoring the need for effective management strategies for aging infrastructure exposed to climate change and extreme hazards (Papathanasiou and Adey 2021; Brighenti et al. 2024).

Many of these structures were designed using empirical or simplified graphical statics methods for gravitational loading, which may be insufficient to address current traffic demands

and extreme events (Oliveira et al. 2010). Slender multi-span MABs are particularly vulnerable to out-of-plane collapse mechanism, typically involving pier base hinge formation and tensile failure at the arch crown (Zampieri et al. 2015, 2021; Cabanzo et al. 2025).

Most existing studies rely on deterministic assessments conducted over discrete capacity or demand levels, which may lead to biased risk estimates (Zampieri et al. 2015, 2021; Da Porto et al. 2016). As uncertainty quantification is computationally demanding, surrogate models (SMs) have been increasingly adopted to accelerate probabilistic evaluations (Tubaldi et al. 2020; Bouzas et al. 2023; Baron et al. 2024; Cabanzo et al. 2025). Nonetheless, most studies have focused on uncertainties in loading and material properties, with only a few applications

explicitly considering geometric variability, typically at the level of individual bridge case studies. In this context, the present study develops a reliability and sensitivity assessment framework for slender multi-span MABs in the Portuguese railway network, explicitly accounting for geometrical uncertainties. Surrogate models are adopted to reduce computational costs in both reliability and sensitivity analyses, and results are compared across different numbers of spans.

2. MABs in the Portuguese railway network

MABs constitute a substantial portion of the Portuguese railway network, with the majority constructed between 1850 and 1950 during a period of rapid railway expansion. At present, MABs represent approximately 30% to 40% of the total bridge stock and are expected to remain in service for the coming decades. Despite their age, recent national initiatives, such as the *Ferrovia2020* program, have primarily focused on track renewal, and several multi-span MABs were reopened without structural strengthening,

replacement, or major repairs. This approach highlights the growing need for performance-based assessments to ensure adequate safety levels for these assets (Silva 2022).

During the late nineteenth century, railway expansion also involved the construction of numerous metallic bridges, including emblematic structures attributed to the *Eiffel* firm (Figure 1) (*Caminhos do ferro e da prata* 2013). By the early twentieth century, increasing train loads and operational speeds raised safety concerns regarding these structures. In southern Portugal, deterioration was further exacerbated by saltwater spillage associated with fish transport, which accelerated corrosion processes. These factors led the railway administration to recognize the need for bridge replacement. Although a nationwide replacement plan was proposed in the 1910s, its implementation was delayed by World War I and economic constraints. As a result, interim measures such as load and speed restrictions were adopted to ensure operational safety (Silva 2022).

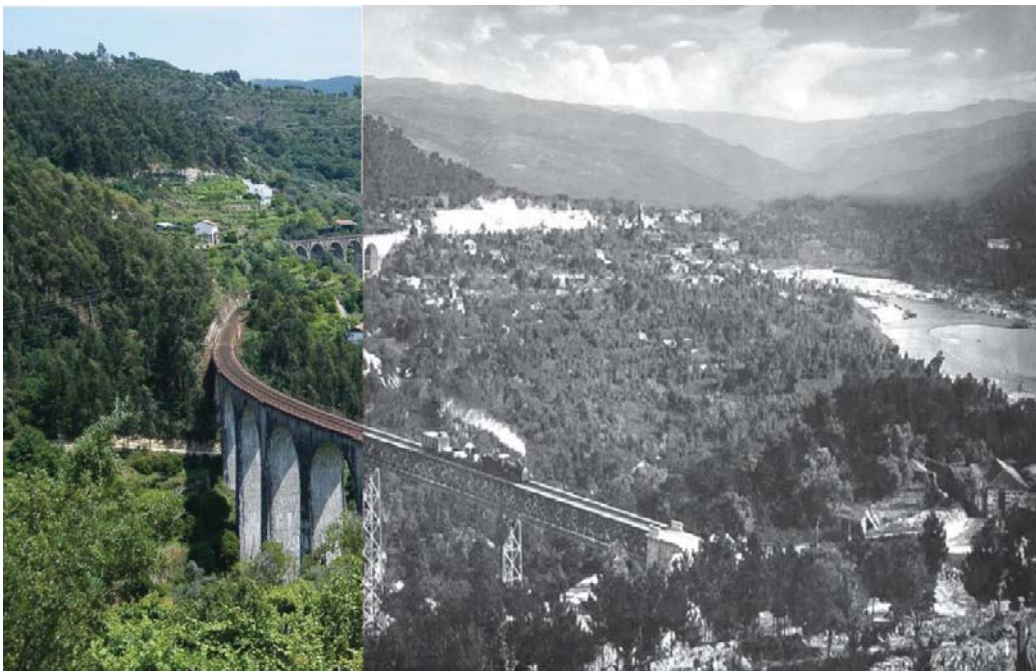


Fig. 1. Composition of the Pala (front) and Ovil (back) viaducts. Adapted from Santos (2011) and *Caminhos do ferro e da prata* (2013).

Replacement works were eventually carried out during the 1930s, including the substitution of 26 steel bridges with MABs in southern Portugal, a region characterized by moderate to high seismicity. Masonry solutions were selected due to their low maintenance requirements and the availability of local stone, despite higher construction costs. Constructed using local labor and materials, these bridges continue to serve key railway corridors for both freight and passenger transport and possess significant heritage value. Notable examples include the *Pala* (PA) and *Ovil* (OV) bridges, which remain prominent features of the Douro Valley landscape (Figure 1).

Portuguese MABs built during the late nineteenth and early twentieth centuries can be classified into two main geometrical configurations. The first group includes bridges with central longer arch spans designed to cross major rivers, such as the *Tâmega* (TA), *Côa* (CO), and *Quinta Nova* (QN) bridges (blue markers in Figure 2). The second group comprises bridges with shorter arches of equal span length (orange markers and grey lines in Figure 2).

Regardless of differences in span length, both groups share common design principles. These include single-track widths of approximately 5 m, spandrel walls with variable thickness increasing toward the base, and piers that taper both longitudinally and transversely with an approximate slope of 1:25. Although the

arches appear semicircular with a nominal constant thickness of 1 m, historical design documents indicate that the arch cross-section is variable, being thinner at the crown and up to twice as thick at the ends (Silva 2022). These empirical design parameters are consistent with those reported for masonry arch bridges in northwest Iberia (Oliveira et al. 2010). Construction typically combines external block masonry with internal infill, often composed of low-quality concrete and irregular masonry.

Despite these shared design characteristics, notable geometric variability exists among the bridges. Differences in the number of spans, span length, total bridge length, and slenderness, quantified by a ratio involving pier height and minimum longitudinal width, illustrate the structural diversity of the Portuguese MAB stock, as shown in Figure 2 and Table A1. Bridges belonging to this group have been previously investigated regarding to the identification of their mechanical and dynamic properties (Neto 2015; Pantò et al. 2024; Cabanzo et al. 2025), as well as seismic fragility assessments incorporating uncertainties in mechanical parameters (Cabanzo et al. 2025; Cabanzo et al. 2025). However, comprehensive information is not available for all bridges constructed during this period. In this context, the present study aims to evaluate the influence of different geometrical configurations on the resulting out-of-plane capacity.

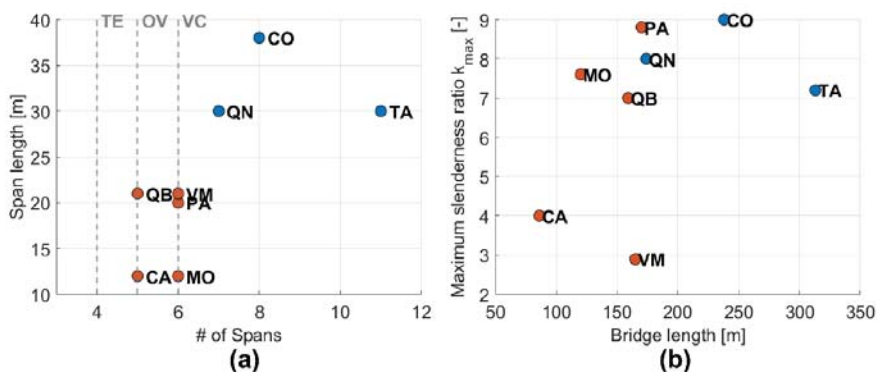


Fig. 2. Sample of Portuguese MABs illustrating the main geometrical differences: (a) number of spans versus span length; (b) total bridge length versus maximum slenderness ratio. Gray lines denote bridges for which only the number of spans is available.

3. Numerical modelling

The finite elements models of the MABs were created using Grasshopper (McNeel 2026), which allows for the parameterization of bridge geometry, including span length, number of spans, slenderness ratio, width, slope, and other key dimensions. This approach allows an accurate representation of the design characteristics shown in the historical MAB drawings. In addition to geometry, material properties as well as boundary conditions, loadings, and convergence criteria, are

implemented directly within Grasshopper, minimizing the need for further manipulation of the models in LUSAS (2023). Figure 3 presents a visualization of a typical MAB alongside the associated Grasshopper script, which contains a total of 408 components.

For illustration, a representative sample of the models is shown in Figure 4. It includes 6 of the 84 models studied in this paper. These models have 6 spans each, with a span length of 25 m. The slenderness ratio for these models varies from 3 (top left corner) to 8 (bottom right corner).

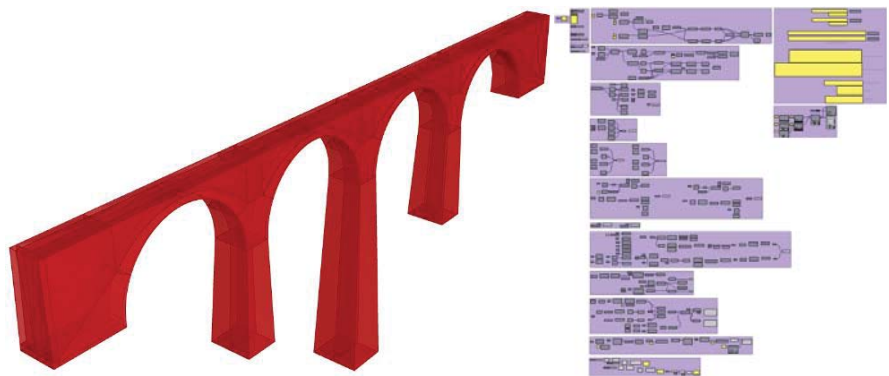


Fig. 3. MAB visualization in Rhino (left) and associated Grasshopper script (right).

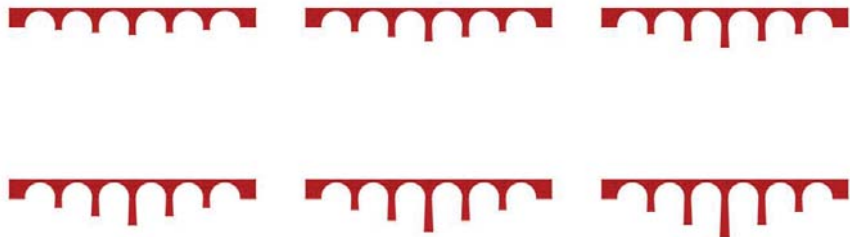


Fig. 4. Sample showing 6 of the 84 models that were studied (number of spans = 6, span length = 25 m, slenderness ratio variable from 3 to 8).

The Grasshopper models are then seamlessly imported into LUSAS for finite element analysis. Solid tetrahedral elements with a mesh size of approximately 1 m were applied. Both the masonry and infill materials are represented using the so-called “smoothed multi-crack” material model, which accounts for both crushing and cracking, as illustrated in Figure 5. Material properties for both

structural masonry and infill concrete are taken from Cabanzo et al. (2025) and summarised in Table 1.

An eigenvalue frequency analysis was first performed to calculate the first lateral mode of the MABs (Figure 6). In a subsequent analysis, an initial load case with gravity was applied. A lateral acceleration was then imposed, scaled according to the previously computed lateral mode shape. A

nonlinear analysis, using modified Newton–Raphson iteration techniques, was carried out to simulate the gradual formation and propagation of cracks in the masonry. The analysis continues until a state of equilibrium can no longer be achieved, at which point the nonlinear procedure fails to converge. This approach constitutes a pseudo-pushover analysis.

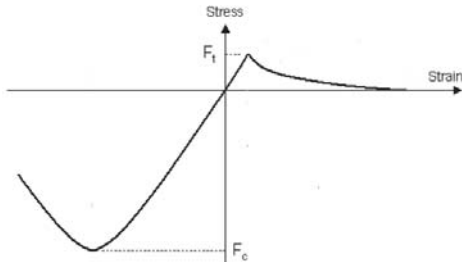


Fig. 5. Stress–strain curve of the smoothed multi-crack model.

Table 1. Deterministic material properties of Portuguese MABs

Property	Masonry	Infill
Young's Modulus [GPa]	20.00	21.50
Compressive strength (F_c) [MPa]	25.00	8.59
Tensile strength (F_t) [MPa]	0.20	1.26

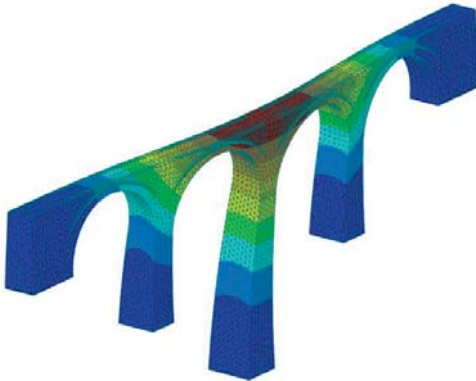


Fig. 6. First lateral natural mode (typical).

4. Reliability analysis

4.1. Variable definition

Based on the **boundaries** observed in Figure 2, the geometrical parameters considered in this study were assumed as continuous uniform distributions within prescribed bounds. **This assumption is justified by the limited information available for the**

complete set of MABs constructed within the scope of this study and aims to capture the full range of possible geometrical configurations for the sensitivity analysis. The analysis focuses on variations in the number of spans, span length, and maximum slenderness ratio. The continuous parameter domains were discretized using a uniform grid-based sampling strategy, summarized in Table 2. The resulting pushover curves for all 84 models are presented in Figure 7.

Table 2. Uniform variables for the geometrical variation of Portuguese MABs

Variable	Upper	Lower	Step
Number of spans (N)	6	4	1
Span length (S) [m]	25	10	5
Slenderness ratio (k_{max})	9	3	1

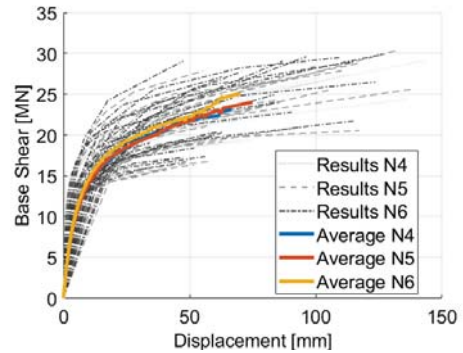


Fig. 7. Resulting pushover curves for bridge groups with the same number of spans (N).

4.2. Surrogate modelling and global sensitivity analysis (GSA)

GSA often becomes computationally demanding for high-dimensional problems due to the large number of model evaluations required. To address this limitation, surrogate models (SMs) have been widely adopted to approximate the numerical response, enabling efficient computation of sensitivity indices while reducing computational cost.

Kriging-based SMs, also known as Gaussian process models, have been extensively employed for GSA of MABs' structural capacity, highlighting their unique ability to capture nonlinear interactions arising from multiple sources of uncertainty. (Tubaldi et al. 2020; Bouzas et al. 2023; Baron et al. 2024; Cabanzo et al. 2025). These models

assume that the structural response is a realization of a Gaussian process indexed by the random input variables. Kriging SMs are then used to derive Sobol' sensitivity indices (Sobol' 2001) to identify the most influential parameters.

In the present study, a Kriging SM was implemented in UQlab (Marelli and Sudret 2014), using a universal trend and an anisotropic ellipsoidal Matérn 5/2 correlation function. For the training of the surrogate model, 90% of the maximum base shear and the corresponding transverse displacement were employed as output variables. A grid-based sampling approach was used to generate training and validation datasets, with an 80/20 split. The leave-one-out (LOO) cross-validation method was applied to assess the surrogate model accuracy. LOO iteratively trains

the model on all but one sample and uses the left-out sample for validation, providing an efficient error estimate, particularly for small datasets. The SM was then used to derive Sobol' sensitivity indices for the output variables (Figure 7).

Figure 8 presents the results of the sensitivity analysis for each span configuration across the sampled parameter combinations. A total of 84 numerical models were generated to train the Kriging surrogate model. The resulting Sobol' indices indicate that span length (S) is the dominant parameter influencing bridge capacity. In contrast, the number of spans (N) exhibits the least influence on the global out-of-plane response. Displacement values show a less clear trend, likely due to the pushover analysis convergence criteria.

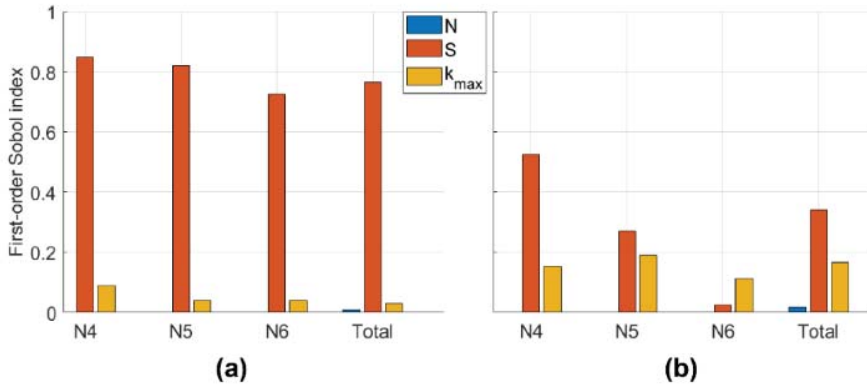


Fig. 8. Results of the GSA illustrating the influence of the main geometrical parameters: (a) 90% maximum base shear; (b) corresponding transversal displacement.

4.3. Reliability assessment using surrogate models

To investigate the influence of the random variables on the estimated structural capacity, a reliability assessment was performed. The objective of this analysis is to derive failure probabilities associated with different limit states defined in terms of maximum base shear. The surrogate models were trained using normalized inputs (standard Gaussian space) and normalized outputs (min-max scaling to the interval [0,1]). Accordingly, the limit states were defined using incremental thresholds with a step size of 5%, resulting in a total of 21 reliability analyses for each group. Importance sampling was adopted to estimate the failure probabilities. This method combines features of First-Order Reliability

Methods (FORM) and Monte Carlo simulation to approximate the design point, while avoiding the need for full convergence of the FORM analysis (Marelli and Sudret 2014; Tabandeh et al. 2022).

Figure 9 presents the results of the reliability analysis, including a log-normal fit of the cumulative distribution function (CDF) of the total capacity for Portuguese multi-span MABs. As expected, the failure probability exhibits a similar overall trend across groups with different numbers of spans. This behavior can be primarily attributed to the dominant influence of span length, which governs most of the variability in the capacity estimates. Nevertheless, at lower capacity levels, a clear correlation with the number of spans is observed. Specifically, for comparable capacity thresholds,

bridges with fewer spans exhibit a higher probability of falling below the prescribed limit.

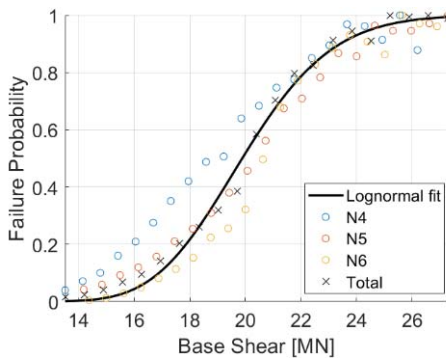


Fig. 9. Results of the reliability analysis, including the log-normal CDF of the total capacity for Portuguese multi-span MABs.

5. Final remarks

This study presented a reliability assessment of the structural performance of multi-span MABs under geometrical uncertainty. By combining the parameterization capabilities of Grasshopper and LUSAS with surrogate models, the proposed approach enabled efficient global sensitivity and reliability analyses while significantly reducing computational cost. The results indicate that span length is the dominant parameter governing out-of-plane capacity, whereas the number of spans plays a secondary role, particularly at lower capacity levels. Reliability analyses further show that bridges with fewer spans are more vulnerable, exhibiting lower base shear. **However, the variation in structural capacity due to geometrical uncertainties is smaller than that arising from uncertainties in the material resistance properties of a single bridge configuration, as demonstrated in Cabanzo et al. (2025) and Cabanzo et al. (2025).** Overall, the proposed assessment methodology provides a practical and scalable tool to support the prioritization, management, and preservation of historical masonry arch bridges within railway networks.

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Appendix A.

This appendix provides additional information on the geometric characteristics of the bridges included in the sample shown in Figure 2, including the bridge name, number of spans (N), span length (S), total length (L), and height (H).

Table A1. Geometrical characteristics of the sampled Portuguese MABs.

Bridge	N	S [m]	L [m]	H [m]
Tâmega - TA	11	30/18	313	60
Côa - CO ^{*,†}	8	38/20	238	56
Quinta Nova - QN ^{*,†}	7	30/12	174	31
Pala - PA [†]	6	20	170	50
Quebradas - QB ^{*,†}	5	21	159	44
Mouratos - MO [†]	6	12	120	30
Vila Meã - VM	6	21	165	26
Calharda - CA	5	12	86	19
Venda Caldas - VC	6	-	-	-
Ovil - OV	5	-	-	-
Teja - TE	4	-	-	-

^{*} Featured in research for material and dynamic calibration and characterization (Neto 2015; Pantò et al. 2024; Cabanzo et al. 2025).

[†] Featured in research considering uncertainties in material properties (Cabanzo et al. 2025; Cabanzo et al. 2025).

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