

A Fuzzy Logic-Based Digital Twin Framework for Risk-Oriented Safety Assessment in Waste Sorting Facilities

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The increasing automation of waste sorting processes brings significant efficiency gains but also introduces new layers of operational uncertainty that further challenge safety management. This paper proposes a conceptual Digital Twin (DT) framework integrating fuzzy logic to support dynamic, risk-oriented safety assessment in automated waste sorting facilities. The novelty of the approach lies in combining Digital Twin-based system representation with fuzzy reasoning, enabling a continuous and interpretable evaluation of safety states under uncertain or incomplete information. The proposed model defines linguistic safety scales (“safe,” “warning,” “hazard”) and a hierarchical fuzzy inference structure, in which sensor-derived indicators, such as temperature, vibration, or current deviations, are aggregated into intermediate risk sub-indices and a global Safety Index. The Digital Twin concept serves as a virtual representation of a cyber-physical sorting line, connecting the physical and cyber layers to provide context-aware insights into operational safety. A simplified proof-of-concept scenario based on a representative cyber-physical sorting system and synthetic input signals demonstrates the model’s capability to detect early warning conditions and dynamically adjust the Safety Index according to the evolving system state. The results indicate that integrating fuzzy reasoning into a DT environment can improve the resilience, reliability, and transparency of safety monitoring, particularly in contexts where data availability is limited or operational variability is high.

Keywords: Digital Twin, fuzzy logic, risk monitoring, waste sorting systems.

1. Introduction

While significantly improving productivity and material recovery in waste management plants (Campana et al., 2025; Giel & Dąbrowska, 2024), higher automation levels introduce new forms of operational uncertainty, related to sensor reliability, environmental conditions, variable material streams, and human-machine interactions (Agnusdei et al., 2021; Douthwaite et al., 2021). These interacting uncertainties may lead to safety-critical situations, such as

overheating, collision risks, or fire hazards, making safety monitoring a key functional layer in automated waste sorting systems (Giel & Dąbrowska, 2025; Zou & Ma, 2025). Conventional safety approaches based on fixed thresholds and deterministic assumptions are often insufficient in such variable and partially observable environments (Agnusdei et al., 2021; Zou & Ma, 2025).

Digital Twin (DT) is a promising technology that, by linking together physical systems with their

virtual representations, may effectively support monitoring and decision-making. Despite its potential, safety-oriented DT applications are still limited, particularly in waste management and circular economy contexts (Campana et al., 2025; Giel & Dąbrowska, 2024, 2025). Moreover, most of the existing DT architectures rely on deterministic models and high-quality data, which may be unrealistic in heterogeneous and uncertain operating conditions typical of waste sorting facilities (Li et al., 2025; Nozari et al., 2025).

To overcome these limitations, this study proposes a conceptual DT framework integrating fuzzy logic for risk-oriented safety assessment in automated waste sorting facilities (Li et al., 2025; Quintanilla et al., 2024). Fuzzy reasoning enables linguistic representation of uncertainty and supports safety evaluation under incomplete or ambiguous data, positioning it as an explicit safety interpretation layer within the DT architecture (Castellani et al., 2021; Quintanilla et al., 2024). This approach enables continuous and interpretable safety assessment instead of binary alarm-based monitoring (Castellani et al., 2021; Li et al., 2025).

The main contributions of this paper are: i) a conceptual DT architecture for dynamic safety monitoring in automated waste sorting systems, ii) the definition of fuzzy linguistic safety scales and inference rules for risk assessment, and iii) a proof-of-concept scenario demonstrating early warning detection under variable operational conditions.

The paper is structured as follows: Section 2 provides the reader with the necessary background on safety issues in waste sorting; Section 3 presents the proposed conceptual framework; Section 4 focuses on fuzzy-reasoning model development; Section 5 describes the proof-of-concept scenario; Section 6 discusses implications and limitations; and Section 7 concludes the paper and outlines future research directions.

2. Theoretical background - Safety and risk in automated waste sorting systems

Waste sorting facilities represent high-risk environments due to the intrinsic variability of processed materials and frequent human-machine interaction (Szulc et al., 2022). In automated sorting systems, risk sources can be broadly

classified into (Levine et al., 2024): process-related risks, technical risks, environmental risks, and human-related risks. Importantly, many hazardous situations are not the result of single critical events, but of the combined and cumulative effects of moderate disturbances, which are difficult to capture using conventional threshold-based safety monitoring approaches.

To address these challenges, modern digital technologies have been increasingly investigated as enablers of advanced safety management. Recent contributions propose DT frameworks for waste stream control, manual sorting management, and system-level monitoring, demonstrating feasibility but also highlighting challenges related to data quality, uncertainty, and limited safety-oriented reasoning capabilities (Giel et al., 2025; Giel & Dąbrowska, 2024; Vargas et al., 2025).

The reviewed literature indicates that safety assessment in waste sorting systems remains largely fragmented and deterministic. This gap highlights the need for an integrated DT-based safety framework that explicitly incorporates uncertainty-aware reasoning mechanisms.

3. Proposed conceptual framework

The proposed framework explicitly accounts for the variability of the material stream, technical degradation of system components, and human-machine interactions under conditions of uncertainty. To address these challenges, the DT is structured into four interconnected layers: the physical layer, cyber model layer, safety inference layer, and decision support layer (Fig. 1).

3.1. Architectural concept of the Digital Twin

The layered architecture enables uncertainty-aware safety assessment and adaptive system response within the unified DT framework.

The physical layer represents the real automated sorting system, including mechanical components, sensors, actuators, the material stream, and the operating environment. It is the primary source of safety risks, resulting from variable material properties (e.g., size, contamination, moisture), technical degradation, environmental conditions, and human operator presence during supervision or intervention. These factors produce heterogeneous and uncertain observations that cannot be reliably assessed using binary safety criteria.

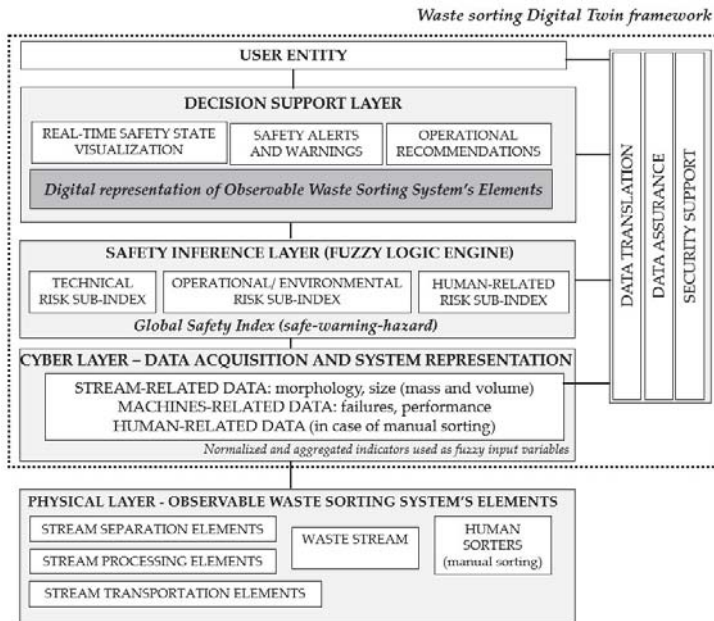


Fig. 1. The conceptual framework of the DT for automated material waste sorting systems. Source: Own contribution based on (Giel et al., 2025)

The cyber model layer provides a digital representation of the physical system by aggregating and processing sensor data into descriptive state variables related to equipment condition, material flow, and environmental context. Due to harsh operating conditions, sensor data are affected by noise, drift, and incompleteness; therefore, this layer delivers an approximate and continuously updated system representation rather than assuming perfect observability. The quality of this representation directly influences the reliability of subsequent safety assessment.

The safety inference layer integrates technical, environmental, material-related, and human-related variables using fuzzy logic reasoning. This approach enables gradual transitions between safety states and aggregation of multiple uncertain or partially conflicting signals into an interpretable safety state or Safety Index, serving as a semantic link between data-driven representation and safety-oriented reasoning. Finally, **the decision support layer** translates inferred safety states into actionable

recommendations, such as operational adjustments, warnings, inspections, or controlled shutdowns. By providing graded and interpretable safety information instead of binary alarms, this layer supports human decision-makers, reduces alarm fatigue, and enables adaptive responses that balance safety and operational continuity.

3.2. Risk identification and integration within the Digital Twin

To enable systematic safety assessment, the proposed DT framework integrates identified risks by explicitly assigning them to specific DT layers and defining their role in the overall safety inference process.

The adopted approach distinguishes between source-level risks (originating in the physical system), information-level risks (related to data acquisition and modeling), and decision-level risks (associated with interpretation and response). This layered risk perspective ensures traceability between observed system behavior, inferred safety states, and resulting operational decisions.

Table 1 summarizes the identified risk categories and their assignment to the corresponding DT layers, along with representative variables and fuzzy safety states used in the inference process. By integrating heterogeneous risk sources across the DT layers, the proposed framework enables a holistic and uncertainty-aware safety assessment. This layered and risk-informed structure forms the basis for the fuzzy safety inference mechanism and the generation of actionable decision support strategies.

Table 1 Mapping of risk categories to Digital Twin layers.

Risk category	DT layer	Representative variables	Fuzzy safety states
Material flow variability	Physical/ Cyber	Material size, flow rate variability, moisture, contamination	<i>low - medium - high</i>
Hazardous material occurrence	Physical/ Safety inference	Temperature anomaly, gas detection, optical irregularities	<i>unlikely - possible - likely</i>
Technical degradation	Physical/ Cyber	Vibration level, bearing temperature, motor current deviation	<i>normal - degraded - failure-prone</i>
Environmental conditions	Physical	Ambient temperature, humidity, dust concentration	<i>acceptable - warning - unsafe</i>
Sensor and data uncertainty	Cyber	Signal noise, missing data ratio, confidence index	<i>reliable - uncertain - unreliable</i>
Human-machine interaction	Safety inference	Operator presence, intervention frequency, proximity	<i>safe - attention - dangerous</i>
Decision and organizational response	Decision support	Alarm response time, procedure compliance	<i>adequate - delayed - insufficient</i>

4. Fuzzy-based safety inference model

The proposed DT framework incorporates a fuzzy logic-based safety inference model. This model operates within the safety inference layer of the DT and integrates technical, environmental,

material-related, and human-machine interaction variables into a unified safety assessment.

4.1. Input variables and linguistic representation

Input variables to the fuzzy inference system (FIS) are derived from the cyber model layer and represent aggregated safety-relevant indicators rather than raw sensor signals. Linguistic representation is therefore employed to capture gradual transitions and imprecise boundaries between operational conditions.

The FIS considers the following representative input variables:

- **material flow condition**, This variable aggregates indicators related to material flow variability, which collectively influence the likelihood of blockages, overloads, and abnormal interactions with system components,
- **technical condition of equipment**, this variable is derived from condition monitoring indicators and represents progressive degradation rather than discrete failure events,
- **environmental operating conditions**, This variable summarizes contextual parameters which may affect both equipment performance and sensor reliability,
- **sensor data reliability**, representing the confidence level of the information provided by the cyber model layer . ,
- **human-machine interaction level**, This variable reflects operator presence in hazardous zones, frequency of manual interventions, and maintenance activities, which introduce dynamic and context-dependent safety risks.

Each input variable is associated with a set of fuzzy membership functions, typically defined using triangular or trapezoidal shapes (Grabisch & Grabisch, 2015; Zimmermann, 2010). Linguistic terms such as low, medium, and high, or normal, degraded, and critical, are used to represent intermediate safety-relevant states without imposing rigid numerical thresholds. This representation enables the FIS to integrate heterogeneous and uncertain inputs in a transparent and interpretable manner.

4.2. Hierarchical fuzzy inference mechanism

Given the presence of five input variables with multiple linguistic states, a direct rule base combining all inputs would lead to an excessive

number of rules, reduced interpretability, and limited scalability.

To address this issue, the proposed model adopts a **hierarchical fuzzy inference structure**, in which related input variables are first aggregated into intermediate risk sub-indices. These sub-indices are subsequently combined to derive a global Safety Index. The hierarchical structure of the model is illustrated in Figure 2, while the definition of sub-indices and their associated inputs is summarized in Table 2.

At the first inference level, input variables are grouped according to their functional role within the automated sorting system and the Digital Twin architecture. Three intermediate fuzzy sub-indices are defined:

- **Process-related Risk Index (PRI)**,
- **Technical Risk Index (TRI)**,
- **Human Interaction Risk Index (HIRI)**.

This grouping reflects the physical and organizational separation of risk sources and allows localized risk escalation to be evaluated independently before global aggregation.

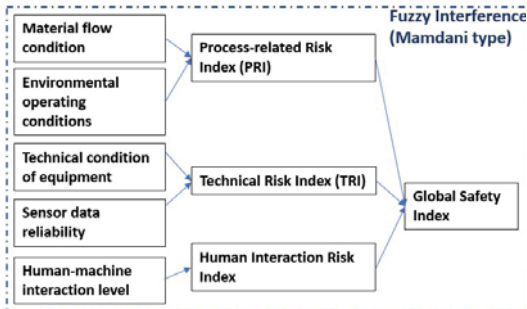


Fig. 2. The proposed hierarchical fuzzy safety inference structure

Each sub-index is inferred using a compact set of fuzzy rules (typically 6 rules for two-input sub-indices and 3 rules for the single-input index), preserving interpretability while capturing nonlinear interactions between risk factors (please add reference here). An exemplary fuzzy rule for the selected sub-indices and the Global Safety Index is presented in Tables 3 and 4. The definition of fuzzy variables and membership functions is given in *Appendix A*.

At the second inference level, the three sub-indices are integrated to derive a **Global Safety Index (GSI)**, which represents the overall operational safety state of the system. This

aggregation captures cross-domain interactions between process conditions, technical health, and human involvement.

Table 2 Definition of fuzzy risk sub-indices.

Sub-index	Input variables	Interpretation	Linguistic states
Process-related Risk Index (PRI)	Material flow condition; Environmental operating conditions	Stability and hazard potential of the technological process	low – medium – high
Technical Risk Index (TRI)	Technical condition of equipment; Sensor data reliability	Equipment health and confidence in technical information	low – medium – high
Human Interaction Risk Index (HIRI)	Human-machine interaction level	Risk related to operator presence and interventions	low – medium – high

Table 3 Representative fuzzy rules for Process-related Risk Index (PRI).

Rule ID	IF Material flow condition	AND Environmental conditions	THEN PRI
PRI-1	low risk	acceptable	low
PRI-2	medium risk	acceptable	medium
PRI-3	medium risk	warning	medium
PRI-4	high risk	acceptable	high
PRI-5	high risk	warning	high
PRI-6	medium risk	unsafe	high

Table 4 Representative fuzzy rules for Global Safety Index.

Rule ID	IF PRI	AND TRI	AND HIRI	THEN Safety Index
GSI-1	low	low	low	safe
GSI-2	medium	low	low	warning
GSI-3	low	medium	medium	warning
GSI-4	medium	medium	low	warning
GSI-5	high	low	low	hazardous
GSI-6	low	high	low	hazardous
GSI-7	medium	high	medium	hazardous
GSI-8	high	medium	high	hazardous

4.3. Output interpretation and linkage to decision support

The output of the hierarchical fuzzy inference model, that is, GSI, may be expressed either as a linguistic safety state (*safe*, *warning*, *hazardous*) or as a continuous numerical value obtained

through defuzzification using the centroid method (Talon & Curt, 2017). The numerical representation enables smooth transitions between safety states and supports graded interpretation of evolving risk conditions, avoiding abrupt changes triggered by isolated threshold violations.

Within the DT framework, the inferred safety output is transferred to the **decision support layer**, where it is mapped to operational responses. Typical actions include adaptive process control (e.g., reducing conveyor speed), operator warnings, inspection or maintenance recommendations, and controlled system shutdowns in high-risk situations.

By separating safety inference from decision execution, the proposed framework ensures transparency and flexibility in safety management.

5. Proof-of-Concept Scenario

To illustrate the applicability of the proposed DT-based fuzzy safety assessment framework, a proof-of-concept (PoC) scenario is developed. The purpose of this scenario is not empirical validation, but conceptual verification of the fuzzy inference mechanism introduced in Section 4 and formally specified in *Appendix A*.

5.1. System model assumptions

The PoC scenario consists of a simplified cyber-physical model of an automated waste sorting line, consistent with the abstract architecture introduced in Section 3. The physical layer is reduced to three core functional elements:

- (i) conveyor-based material transport,
- (ii) sorting and handling operations, and
- (iii) operator interaction zones.

To ensure conceptual generality and reproducibility, the PoC relies on synthetic input signals representing typical operational patterns in waste sorting facilities, including gradual equipment degradation, material contamination and flow irregularities, environmental disturbances, and varying levels of human-machine interaction. All signals evolve continuously rather than abruptly, reflecting the assumption embedded in the fuzzy model that hazardous situations often arise from the accumulation of moderate disturbances.

All inputs are normalized and mapped to the linguistic variables defined in Section 4.1, using the membership functions specified in *Appendix*

A (Tables A1–A3). The resulting fuzzy inputs constitute the cyber-layer representation of the system state within the Digital Twin.

5.2. Implementation logic (conceptual only)

The PoC implementation follows the hierarchical fuzzy inference architecture defined in Section 4.2 and illustrated in Figure 2. Five aggregated fuzzy input variables, corresponding to the linguistic inputs specified in Section 4.1 and *Appendix A*, are processed through a two-stage inference mechanism.

5.3. Example results and interpretation

The behavior of the fuzzy safety assessment model is illustrated using three time-dependent operational scenarios, each corresponding to different combinations of fuzzy inputs and rule activations.

The temporal evolution of GSI across the three scenarios is illustrated in Figure 3, showing smooth transitions between linguistic safety states rather than abrupt changes.

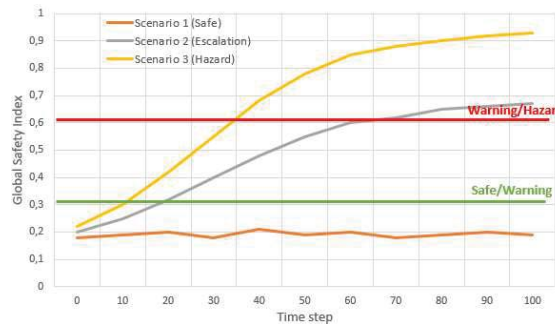


Fig. 3. GSI evolution for all three scenarios

In the PoC simulation, the discrete timesteps shown in Figure 3 represent short-term operational intervals on the order of seconds. This temporal resolution reflects the typical update rates of industrial monitoring systems and control loops in automated sorting lines, where sensor signals such as vibration, current, or temperature are sampled continuously and processed at sub-minute scales. The continuous evolution of the synthetic signals over consecutive timesteps therefore represents gradual changes in system condition over tens to hundreds of seconds, rather than abrupt step changes, consistent with the assumption that hazardous situations often emerge from accumulating moderate disturbances.

6. Discussion

The central strength of the proposed framework is its ability to integrate heterogeneous risk sources, explicitly defined in the fuzzy model (Section 4, Appendix A), including material flow conditions, technical degradation, environmental influences, sensor reliability, and human-machine interaction. Unlike conventional safety monitoring systems that assess these factors independently, the fuzzy inference mechanism captures their combined and nonlinear effects.

The hierarchical aggregation of inputs into intermediate risk sub-indices reduces rule complexity while preserving causal interpretability. This design enhances practical applicability, as explainability is essential for operator trust and regulatory acceptance.

From a DT perspective, the framework extends DT functionality beyond passive monitoring toward proactive safety support. As demonstrated in Section 5, the continuous Safety Index enables early warning detection and supports adaptive decision-making, contributing to improved system resilience through anticipation rather than reaction to safety threats.

However, several limitations remain. The PoC draws on simplified assumptions and synthetic data, limiting direct generalization to industrial installations. The fuzzy membership functions and rules were defined conceptually and have not been empirically calibrated. Moreover, the framework currently focuses on safety assessment and decision support without integration of predictive maintenance or automated control. These limitations reflect the conceptual scope of the study and outline clear directions for future research.

7. Conclusion

This paper proposed a conceptual DT-based framework integrating fuzzy logic for dynamic, risk-oriented safety assessment in automated waste sorting systems.

The main contribution is a multi-layer DT architecture combining physical, cyber, safety inference, and decision support layers, enabling continuous and interpretable safety evaluation under uncertainty. A hierarchical fuzzy model aggregates heterogeneous risk factors into a single Safety Index, avoiding rigid deterministic thresholds.

A PoC scenario based on a simplified cyber-physical sorting line and synthetic data

demonstrated the logical consistency and dynamic behavior of the framework. The results show the model's ability to identify early warning conditions caused by accumulated moderate disturbances and combined technical and human-related risks, confirming the feasibility of fuzzy reasoning within a Digital Twin safety context. Future studies will focus on validation using real operational data, calibration of fuzzy parameters, and integration with predictive maintenance and anomaly detection methods.

Appendix A. Definition of fuzzy variables and membership functions

This appendix provides supplementary information regarding the definition of fuzzy input variables, sub-indices, and the output Safety Index used in Section 4.

A.1. Input variables and normalization

All input variables are normalized to a common range [0,1]. Normalization is performed at the cyber model layer using min-max scaling or expert-defined reference ranges.

A.2. Membership functions for input variables

Triangular and trapezoidal membership functions are employed due to their simplicity, interpretability, and low computational cost. Overlapping membership functions are used to allow smooth transitions between linguistic states.

A.2.1. Material flow condition

Linguistic term	Membership function	Normalized range
Low risk	Trapezoidal	[0.0, 0.0, 0.2, 0.4]
Medium risk	Triangular	[0.3, 0.5, 0.7]
High risk	Trapezoidal	[0.6, 0.8, 1.0, 1.0]

A.2.2. Technical condition of equipment

Linguistic term	Membership function	Normalized range
Normal	Trapezoidal	[0.0, 0.0, 0.25, 0.45]
Degraded	Triangular	[0.35, 0.55, 0.75]
Critical	Trapezoidal	[0.65, 0.85, 1.0, 1.0]

A.2.3. Environmental operating conditions

Linguistic term	Membership function	Normalized range
Acceptable	Trapezoidal	[0.0, 0.0, 0.3, 0.5]
Warning	Triangular	[0.4, 0.6, 0.8]
Unsafe	Trapezoidal	[0.7, 0.9, 1.0, 1.0]

A.2.4. Sensor data reliability

Linguistic term	Membership function	Normalized range
Reliable	Trapezoidal	[0.0, 0.0, 0.3, 0.5]
Uncertain	Triangular	[0.4, 0.6, 0.8]
Unreliable	Trapezoidal	[0.7, 0.9, 1.0, 1.0]

A.2.5. Human-machine interaction level

Linguistic term	Membership function	Normalized range
Low	Trapezoidal	[0.0, 0.0, 0.25, 0.45]
Medium	Triangular	[0.35, 0.55, 0.75]
High	Trapezoidal	[0.65, 0.85, 1.0, 1.0]

A.3. Membership functions for sub-indices

The three intermediate risk sub-indices (PRI, TRI, HIRI) are represented using identical linguistic structures to ensure interpretability and comparability.

Linguistic term	Membership function	Normalized range
Low	Trapezoidal	[0.0, 0.0, 0.3, 0.5]
Medium	Triangular	[0.4, 0.6, 0.8]
High	Trapezoidal	[0.7, 0.9, 1.0, 1.0]

A.4. Output variable: Global Safety Index

The output of the hierarchical fuzzy inference model is GSI, defined over the normalized range [0,1]. Three linguistic safety states are used:

Safety state	Membership function	Normalized range
Safe	Trapezoidal	[0.0, 0.0, 0.25, 0.45]
Warning	Triangular	[0.35, 0.55, 0.75]
Hazardous	Trapezoidal	[0.65, 0.85, 1.0, 1.0]

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