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Sensitivity and uncertainty propagation analysis of RC beams shear strength using polynomial chaos expansion method.

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In the paper, a shear strength probabilistic assessment of concrete beams reinforced with steel bars is performed. The shear strength is analyzed by modelling two approaches. One is the low-fidelity models, which are analytical formulas based on the fracture mechanics and semi-empirical mechanics-based mode. The other is based on the numerical simulations performed using the advanced software (Atena) based on non-linear mechanics principles. The aim of the analysis is to perform the uncertainty propagation, sensitivity analysis and model uncertainty assessment. The stochastic analysis is made using polynomial chaos expansion method. The application of this method and the analysis of its performance is the main novelty introduced in the paper.

Keywords: RC beams, shear strength, stochastic analysis, structural safety, polynomial chaos expansion, Sobol's indices, probability density function.

1. Introduction

Shear strength might be analyzed using low-fidelity models: simple analytical formulas based on the fracture mechanics and semi-empirical mechanics-based mode, or using advanced finite element based software, which are dedicated for the non-linear mechanical problems of composited, i.e. Atena, Abaqus, etc. In the presented manuscript the uncertainty propagation and sensitivity of the shear strength was analyzed using both of approaches. In the first problem the shear strength was calculated using simple formulas taken based on Eurocode 2 EN 1992-1-1. In the second approach the Atena software was used to perfume the mechanical analysis of the RC beams. Then the polynomial chaos expansion method was applied to investigate the uncertainty propagation and sensitivity analysis. To reach the goal the kernel function was applied to determine the probability density function and the Sobol's indices were calculated for the sensitivity analysis. The application of polynomial chaos expansion method is the main novelty introduced in the paper. The analysis of the PCE performance for shear strength stochastic analysis is presented in the manuscript.

2. Shear strength of RC beams

Shear is a complex phenomenon in reinforced concrete beams without transverse reinforcement, depending on several interacting mechanisms. Concrete carries part of the shear through tensile stresses in the cracked zone, influenced by its tensile strength and crack width (Yu et al. 2022). Additional contributions come from aggregate interlock and the dowel action of longitudinal bars, which help transfer shear forces across cracks. Shear capacity is also affected by the concrete compressive strength, longitudinal reinforcement ratio, effective depth, and size effects, which reduce strength in larger beams. Different design codes and guidelines account for these parameters and physical mechanisms in beams under load in varying ways. The Eurocode 2 EN 1992-1-1 provides the equation to calculate the maximum shear strength of concrete beams with the longitudinal reinforcement and without transversal ones, which reads:

$$V_{Rd,c} = C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} \cdot b_w \cdot d \qquad (1)$$

where is b_w the width of the beam cross-section, d is the effective depth of the beam ($d = h - c - \phi/2$, h is the height of the beam and c is the thickness of concrete cover and ϕ is the diameter of longitudinal reinforcement), f_{ck} is the compressive strength of the concrete, $C_{Rd,c}$ is the empirical coefficient for concrete shear resistance, which might be calculated by the equation:

$$C_{Rd,c} = \frac{0.18}{\gamma_c} \qquad (2)$$

where γ_c is the partial safety factor of concrete. The term $(100 \cdot \rho_l \cdot f_{ck})^{1/3}$ in Eq. (1) represents the limitation of crack opening by the longitudinal reinforcement and contribution of concrete to shear stress transfer. The longitudinal reinforcement ratio ρ_l is defined as:

$$\rho_l = min\left(\frac{A_{sl}}{b_w d}; 0.02\right) \qquad (3)$$

where A_{sl} is the cross-section area of tensile longitudinal reinforcing bars. From Eq. (3) it results that if the longitudinal reinforcement ratio cannot be larger than 0.02 despite the real area of the provided tensile reinforcement. The size effect factor k, is defined as:

$$k = min\left(1 + \sqrt{\frac{200}{d}}; 2.0\right) \qquad (4)$$

where d is the effective depth of the beam.

The shear strength can also be calculated using the numerical software. Here the most popular numerical method is FEM, where the analysed domain is divided into the final number of elements. The shear strength will be analysed using Atena software.

3. Stochastic model

3.1. *Random input variable*

For most of the engineering problem, particularly related to the structure reliability assessment, instead of deterministic numbers one has to provide the probability analysis. Therefore both the input vector and output have to be considered as the random variables. The output is the calculated sear strength of the beam, defined by the Eq.(1). As inputs one have to considered the number of variables, namely: f_{ck}, the effective dimensions of the beam: d and b_w, A_{sl}. According to the engineering practice and literature (Shimiziu et al. 2010, Nowak et al. 2011) the compressive strength of concrete varies and the its coefficient of variance may range from 0.05 to 0.2. The dimensions of the beam h and b_w do not vary for precast elements and vary only slightly for site-cast concrete beams for thorough on-site construction supervision. Similarly, the diameters of the longitudinal bars are constant and deviations are controlled during the production stage. However, during placement of the concrete mix the bars spacers may become displaced, leading to variations in the thickness of the concrete cover. The experiments performed by the authors revealed that the deviation of c does not depend on c_{mean}. The standard deviation of c is equal to approx.. 5 mm, regardless of its design value. Consequently in the model, Eq. (1), one should consider two input random variables, namely the compressive strength (f_{ck}) and the effective depth (d) or alternatively thickness of concrete cover c.

3.2. *Model for uncertainties assessment*

There are numbers of approaches, which may be applied for uncertainty assessment, i.e. Baghi et al. 2018, Sykora et al. 2018. However in this work the Polynomial Chaos Expansion (PCE) model is used for analysis (Sudret 2008). Employing PCE the output random variable is approximated by the polynomials, according to the equation:

$$V_{Rd,c} \approx V_{Rd,c}^{PCE} = \sum_{i=1}^{\infty} y_i \Psi_i(\boldsymbol{X}) \quad (5)$$

where $\boldsymbol{X}$ is the normalize random input vector, Ψ_i are the multivariate orthonormal polynomials (basis fuctions) and y_i are coefficients. The multivariate orthonormal polynomials are the products of univariate polynomials, which are chosen with regard to the properties of the input random variables, i.e. for normally distributed input the Hermite polynomials must be taken, while for uniformly the Legendre. Since in our case both compressive strength and effective depth are normally distributed the Hermite polynomials are applied. They may be generated according to the rule:

$$He_0(x) = 1, He_1(x) = x, \quad (6)$$

$$He_{n+1}(x) = xHe_n(x) - nHe_{n-1}(x)$$

The details of PCE approximation can be found in [Sudret 2008]. The key point of the approximation is to find the coefficients y_i, see Eq. (5). It can be done by least square minimization based on model (1) calculations of $V_{Rd,c,k}^{PCE}$ for the finite number of sampling points: $\left(\overline{f_{ck}}, \bar{d}\,\right)_k$, where $\overline{f_{ck}}$ is normalized concrete compressive strength and $\bar{d}$ is the normalized effective depth. The vector $\boldsymbol{Y}$ containing coefficients y_i may be calculated from equation:

$$\boldsymbol{Y} = \left(\boldsymbol{A}^T\boldsymbol{A}\right)^{-1}\boldsymbol{A}^T\boldsymbol{V}_{Rd,c}^{PCE} \quad (7)$$

where $\boldsymbol{A}$ is the experimental matrix.

Polynomial series expansion can be treated as the response surface for the problem. By Monte Carlo simulation with the Latin Hypercube Sampling (LHS) one can derive the probability distribution of output (shear strength) by means of kernel function:

$$f_{Vrd,c}(y) = \frac{1}{n \cdot h} \cdot \sum_{j=1}^{n} Ker\left(\frac{y - V_{Rd,c,j}^{PCE}}{h}\right) \quad (8)$$

where h is bandwidth and n is the number of numerical samples. The kernel function may be taken as:

$$Ker(t) = \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}}. \quad (9)$$

The influence of input random variable (compressive strength and effective depth) on shear strength can be estimated by sensitivity analysis, i.e. by calculating Sobol's indices (Sobol 2001). Because the basis functions are orthonormal the statistical moments can be calculated by means of coefficients y_i, taken from Eq. (5):

$$\mu_{Vrd,c} = y_0$$

$$Var\left[V_{Rd,c}\right] = \sigma_{Vrd,c}^2 = \sum_{u \in \{f_{ck}, d\}} D_u$$

$$D_u = \sum_{\alpha \in \Delta_u} y_\alpha \quad (10)$$

where Δ_u contains all terms related to the given input random variable f_{ck}, d.

3.2. *Results – code model*

In the first part let us calculate the probability density function of shear strength calculated according to Eurocode, Eq. (1) and do the sensitivity analysis. As mentioned earlier the random input variables are compressive strength and effective depth. It was assumed that both input random variable have normal distribution and are independent: $f_{f_{ck}} \sim \mathcal{N}(25{,}5)$ and $f_d \sim \mathcal{N}(400{,}5)$, the strength is given in MPa and dimensions in mm, $f_{f_{ck} \times d} = f_{f_{ck}} \cdot f_d$. The constant input parameters were assumed as $b_w = 250mm$, 3 bars with diameter 16 mm were taken as longitudinal reinforcement, the partial safety factor $\gamma_c = 1.4$. The maximum order of approximating polynomial was taken as 3. First, the parameters related to approximating polynomials, Eq. (5) were calculated using least square method assuming 100 sampling 2D points $\left(\overline{f_{ck}}, \overline{d}\right)_j$. The sampling points were generated using LHS assuming normal distribution $\mathcal{N}(0{,}1)$ of both (normalized) variables. The isoprobabilistic transform was used to link the real random variable with the normalized ones. Then, the probability density function of shear strength and Sobol's indices were calculated. After least square error minimization the PCE approximation Eq. (5) one obtained the response surface of shear strength, see Fig. 1, as a function of two input random variable: compressive strength and effective depth. For the reader's convenience the arguments of response surface (shear strength) were scaled to their real values. The maximum order of multivariate polynomials was assumed as three.

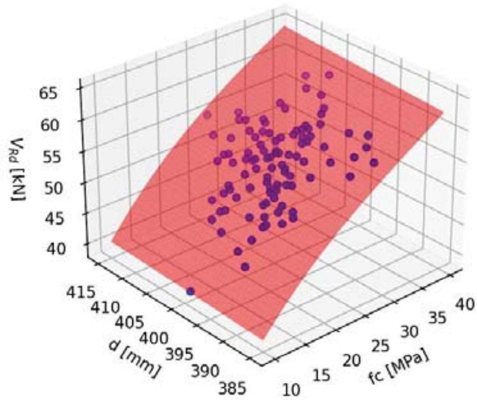

Fig. 1. The approximation of shear strength using polynomial chaos expansion (polynomial order is equal to 3).

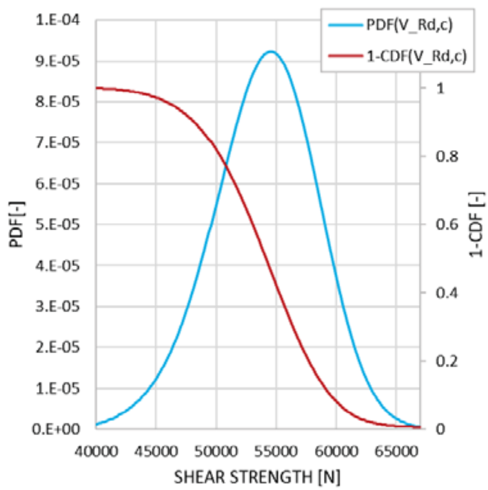

Fig. 2. The probability density function and cumulative density function of shear strength assuming two input random variable: compressive strength, effective depth.

Fig. 2 presents the PDF and CDF of shear strength. To make the results user friendly (in civil The lower the design load-bearing capacity of the structure, the greater the safety margin) we decided to plot 1-CDF($V_{Rd,c}$). One can notice that despite the fact that the input variables have normal distribution the PDF of output strength is asymmetric. The global sensitivity of shear strength was analysed by calculating the Sobol's indices according to Eq. (10). The results are presented in Table 1.

Table 1. The Sobol's indices calculated for shear strength.

	Sobol's indices	
	First order	Total order
f_{ck}	0.9930	0.9933
d	0.0069	0.0071

One can notice that the variance of compressive strength has much larger effect on the variance of shear strength than that of the effective depth, for the assumed distributions of inputs.

3.2. *Results – Atena simulations*

The second investigated problem was related to the shear strength of simply supported beam again without transversal reinforcement. In this problem the response surface was calculated by means of 75 results of numerical simulation of the entire beam. The calculations were performed using FEM software Atena. The span of the beam was equal to 2.4m, the width and height were assumed as in the previous problem: $b_w = 250mm$ and $d = 400mm$ $(d = h - c - \phi/2)$. The input random variables were, like for the previous case, the compressive strength $f_{f_{ck}} \sim \mathcal{N}(60,6)$ and thickness of the concrete cover: $f_c \sim \mathcal{N}(30,5)$, the strength is given in MPa and dimensions in mm For this problem the response surface was approximated based on 75 numerical simulations, employing the professional FEM software. The maximum order of multivariate polynomials was assumed as three. The response surface is presented in Fig. 3.

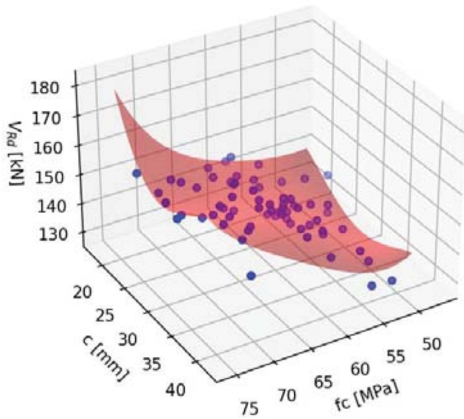

Fig. 3. The approximation of shear strength using polynomial chaos expansion (polynomial order is equal to 3).

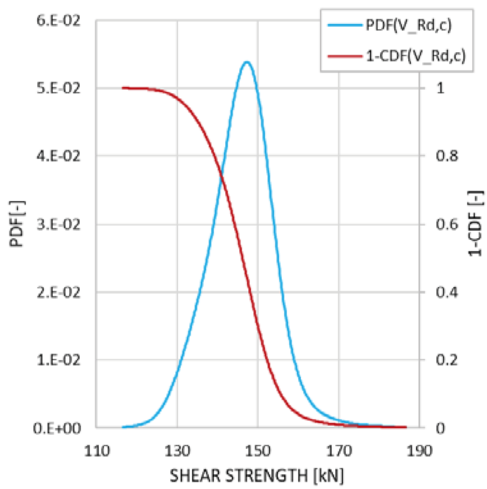

Fig. 4. The probability density function and cumulative density function of shear strength assuming two input random variable: compressive strength, effective depth.

Fig. 4 presents the PDF and CDF of shear strength obtained based on the calculation performed by Atena software. To make the results user friendly (in civil One can notice that despite the fact that the input variables have normal distribution the PDF of output strength is asymmetric, see Fig. 4.

The analysis of shear strength sensitivity was performed by calculating the Sobol's indices according to Eq. (10). The results are presented in Table 2.

Table 2. The Sobol's indices calculated for shear strength.

	Sobol's indices	
	First order	Total order
f_{ck}	0.8161	0.9057
d	0.0933	0.1828

Conclusions

The stochastic analysis is of the critical importance when reliability of the structures is assessed. The presented analysis concerned the uncertainty propagation and sensitivity analysis of the shear strength of RC beams. No transverse reinforcement was assumed for the analysed members. It was reviled that the application of polynomial chaos expansion are efficient for stochastic analysis, which based on both analytical code-based formulas and results of numerical simulations.

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References

EN 1992-1-1 (2004) Eurocode 2: Design of Concrete Structures – Part 1-1: General Rules and Rules for Buildings. Brussels: European Committee for Standardization. (CEN).

Yu, Q.; Ruiz, M.F.; Muttoni, A. (2022) Considerations on the partial safety factor format for reinforced concrete structures accounting for multiple failure modes. Engineering Structures 264, pp. 114442.

Shimiziu, Y.; Hirosawa, M.; Zhou, J. (2000) Statistical analysis of concrete strength in existing reinforced concrete buildings in Japan. 12th World conference on earthquake engineering, Auckland, New Zealand.

Nowak, A.; Rakoczy, A.; Szeliga, E.K. (2011) Revised statistical resistance models for R/C structural components. Andy Scanlon Symposium on Serviceability and Safety of Concrete Structures: From Research to Practice. American Concrete Institute, ACI Special Publication 284, pp. 61–76.

Baghi, H.; Barros, J.A.; Kaszubska, M.; Kotynia, R. (2018) Shear behavior of concrete beams reinforced exclusively with longitudinal glass fiber reinforce polymer bars: Analytical model. Structural Concrete 19, pp. 162–173.

Sýkora, M.; Krejsa, J.; Mlčoch, J.; Prieto, M.; Tanner, P. (2018) Uncertainty in shear resistance models of reinforced concrete beams according to fib MC2010. Structural Concrete 19, pp. 284–295.

Sobol, I.M., (2001) Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates, Mathematics and Computers in Simulation 55, 271–280.

Sudret, B., (2008) Global sensitivity analysis using polynomial chaos expansions, Reliability Engineering and System Safety 93, 964–979